

TOTAL ROMAN REINFORCEMENT IN GRAPHS

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Abstract

A total Roman dominating function on a graph G is a labeling $f : V(G) \rightarrow \{0, 1, 2\}$ such that every vertex with label 0 has a neighbor with label 2 and the subgraph of G induced by the set of all vertices of positive weight has no isolated vertex. The minimum weight of a total Roman dominating function on a graph G is called the total Roman domination number of G . The total Roman reinforcement number $r_{tR}(G)$ of a graph G is the minimum number of edges that must be added to G in order to decrease the total Roman domination number. In this paper, we investigate the properties of total Roman reinforcement number in graphs, and we present some sharp bounds for $r_{tR}(G)$. Moreover, we show that the decision problem for total Roman reinforcement is NP-hard for bipartite graphs.

Keywords: total Roman domination number, total Roman reinforcement number.

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REFERENCES

- [1] H. Abdollahzadeh Ahangar, J. Amjadi, S.M. Sheikholeslami and M. Soroudi, *On the total Roman domination number of graphs*, *Ars Combin.*, to appear.
- [2] H. Abdollahzadeh Ahangar, M.A. Henning, V. Samodivkin and I.G. Yero, *Total Roman domination in graphs*, *Appl. Anal. Discrete Math.* **10** (2016) 501–517.
doi:10.2298/AADM160802017A
- [3] J. Amjadi, S. Nazari-Moghaddam and S.M. Sheikholeslami, *Global total Roman domination in graphs*, *Discrete Math. Algorithms Appl.* **9** (2017) ID: 1750050.
doi:10.1142/S1793830917500501
- [4] J. Amjadi, S.M. Sheikholeslami and M. Soroudi, *Nordhaus-Gaddum bounds for total Roman domination*, *J. Comb. Optim.* **35** (2018) 126–133.
doi:10.1007/s10878-017-0158-5
- [5] J. Amjadi, S. Nazari-Moghaddam and S.M. Sheikholeslami and L. Volkmann, *Total Roman domination number of trees*, *Australas. J. Combin.* **69** (2017) 271–285.
- [6] E.J. Cockayne, R.M. Dawes and S.T. Hedetniemi, *Total domination in graphs*, *Networks* **10** (1980) 211–219.
doi:10.1002/net.3230100304
- [7] E.J. Cockayne, P.A. Dreyer Jr., S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*, *Discrete Math.* **278** (2004) 11–22.
doi:10.1016/j.disc.2003.06.004
- [8] O. Favaron, H. Karami, R. Khoelilar, and S.M. Sheikholeslami, *On the Roman domination number of a graph*, *Discrete Math.* **309** (2009) 3447–3451.
doi:10.1016/j.disc.2008.09.043
- [9] M.R. Garey and D.S. Johnson, *Computers and Intractability: A Guide to the Theory of NP-Completeness*, (Freeman, San Francisco, 1979).
- [10] M.A. Henning, *A survey on selected recent results on total domination in graphs*, *Discrete Math.* **309** (2009) 32–63.
doi:10.1016/j.disc.2007.12.044
- [11] M.A. Henning, N. Jafari Rad and J. Raczek, *A note on total reinforcement in graphs*, *Discrete Appl. Math.* **159** (2011), 1443–1446.
doi:10.1016/j.dam.2011.04.024
- [12] M.A. Henning and A. Yeo, *Total Domination in Graphs* (Springer, New York, 2013).
doi:10.1007/978-1-4614-6525-6
- [13] N. Jafari Rad and S. Sheikholeslami, *Roman reinforcement in graphs*, *Bull. Inst. Combin. Appl.* **61** (2011) 81–90.
- [14] C.-H. Liu and G.J. Chang, *Roman domination on 2-connected graphs*, *SIAM J. Discrete Math.* **26** (2012) 193–205.
doi:10.1137/080733085

- [15] C.-H. Liu and G.J. Chang, *Upper bounds on Roman domination numbers of graphs*, Discrete Math. **312** (2012) 1386–1391.
doi:10.1016/j.disc.2011.12.021
- [16] C.-H. Liu and G.J. Chang, *Roman domination on strongly chordal graphs*, J. Comb. Optim. **26** (2013) 608–619.
doi:10.1007/s10878-012-9482-y
- [17] P. Pavlič and J. Žerovnik, *Roman domination number of the Cartesian products of paths and cycles*, Electron. J. Combin. **19** (3) (2012) #P19.
- [18] C.S. Revelle and K.E. Rosing, *Defendens Imperium Romanum: a classical problem in military strategy*, Amer. Math. Monthly **107** (2000) 585–594.
doi:10.1080/00029890.2000.12005243
- [19] N. Sridharan, M.D. Elias and V.S.A. Subramanian, *Total reinforcement number of a graph*, AKCE Int. J. Graphs Comb. **4** (2007) 197–202.
- [20] I. Stewart, *Defend the Roman Empire*, Sci. Amer. **281** (1999) 136–139.
doi:10.1038/scientificamerican1299-136
- [21] I.G. Yero and J.A. Rodríguez-Velázquez, *Roman domination in Cartesian product graphs and strong product graphs*, Appl. Anal. Discrete Math. **7** (2013) 262–274.
doi:10.2298/AADM130813017G

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