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THE TURÁN NUMBER OF $2P_7$

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Abstract

The Turán number of a graph H, denoted by $ex(n, H)$, is the maximum number of edges in any graph on n vertices which does not contain H as a subgraph. Let P_k denote the path on k vertices and let mP_k denote m disjoint copies of P_k. Bushaw and Kettle [*Turán numbers of multiple paths and equibipartite forests*, Combin. Probab. Comput. 20 (2011) 837–853] determined the exact value of $ex(n, kP_\ell)$ for large values of n. Yuan and Zhang [*The Turán number of disjoint copies of paths*, Discrete Math. 340 (2017) 132–139] completely determined the value of $ex(n, kP_3)$ for all n, and also determined $ex(n, F_m)$, where F_m is the disjoint union of m paths containing at most one odd path. They also determined the exact value of $ex(n, P_3 \cup P_{2\ell+1})$ for $n \geq 2\ell + 4$. Recently, Bielak and Kieliszek [*The Turán number of the graph* $2P_5$, Discuss. Math. Graph Theory 36 (2016) 683–694], Yuan and Zhang [*Turán numbers for disjoint paths*, arXiv:1611.00981v1] independently determined the exact value of $ex(n, 2P_5)$. In this paper, we show that $ex(n, 2P_7) = \max\{[n, 14, 7], 5n - 14\}$ for all $n \geq 14$, where $[n, 14, 7] = (5n + 91 + r(r - 6))/2$, $n - 13 \equiv r \pmod 6$ and $0 \leq r < 6$.

Keywords: Turán number, extremal graphs, $2P_7$.

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