

ON DECOMPOSING THE COMPLETE SYMMETRIC DIGRAPH INTO ORIENTATIONS OF $K_4 - e$

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Abstract

Let D be any of the 10 digraphs obtained by orienting the edges of $K_4 - e$. We establish necessary and sufficient conditions for the existence of a (K_n^*, D) -design for 8 of these digraphs. Partial results as well as some nonexistence results are established for the remaining 2 digraphs.

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