

DOMINATION SUBDIVISION AND DOMINATION MULTISUBDIVISION NUMBERS OF GRAPHS

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Abstract

The *domination subdivision number* $sd(G)$ of a graph G is the minimum number of edges that must be subdivided (where an edge can be subdivided at most once) in order to increase the domination number of G . It has been shown [10] that $sd(T) \leq 3$ for any tree T . We prove that the decision problem of the domination subdivision number is NP-complete even for bipartite graphs. For this reason we define the *domination multisubdivision number* of a nonempty graph G as a minimum positive integer k such that there exists an edge which must be subdivided k times to increase the domination number of G . We show that $msd(G) \leq 3$ for any graph G . The domination subdivision number and the domination multisubdivision number of a graph are incomparable in general, but we show that for trees these two parameters are equal. We also determine the domination multisubdivision number for some classes of graphs.

Keywords: domination, domination subdivision number, domination multisubdivision number, trees, computational complexity.

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