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ON INDEPENDENT DOMINATION IN PLANAR CUBIC GRAPHS

Gholamreza Abrishami [a]

Michael A. Henning [b,1]

and

Freydoon Rahbarnia[a,2]

[a] *Department of Applied Mathematics*
Ferdowsi University of Mashhad
P.O. Box 1159, Mashhad 91775, Iran

[b] *Department of Pure and Applied Mathematics*
University of Johannesburg
Auckland Park, 2006 South Africa

e-mail: gh.abrishamimoghadam@mail.um.ac.ir
mahenning@uj.ac.za
rahbarnia@um.ac.ir

Abstract

A set S of vertices in a graph G is an independent dominating set of G if S is an independent set and every vertex not in S is adjacent to a vertex in S. The independent domination number, $i(G)$, of G is the minimum cardinality of an independent dominating set. Goddard and Henning [Discrete Math. 313 (2013) 839–854] posed the conjecture that if $G \notin \{K_{3,3}, C_5 \,\square\, K_2\}$ is a connected, cubic graph on n vertices, then $i(G) \le \frac{3}{8}n$, where $C_5 \,\square\, K_2$ is the 5-prism. As an application of known result, we observe that this conjecture is true when G is 2-connected and planar, and we provide an infinite family of such graphs that achieve the bound. We conjecture that if G is a bipartite, planar, cubic graph of order n, then $i(G) \le \frac{1}{3}n$, and we provide an infinite family of such graphs that achieve this bound.

Keywords: independent domination number, domination number, cubic graphs.

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[2]Corresponding author.

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