PANCYCLICITY WHEN EACH CYCLE CONTAINS $k$ CHORDS

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Abstract

For integers $n \geq k \geq 2$, let $c(n,k)$ be the minimum number of chords that must be added to a cycle of length $n$ so that the resulting graph has the property that for every $l \in \{k, k+1, \ldots, n\}$, there is a cycle of length $l$ that contains exactly $k$ of the added chords. Affif Chaouche, Rutherford, and Whitty introduced the function $c(n,k)$. They showed that for every integer $k \geq 2$, $c(n,k) \geq \Omega_k(n^{1/k})$ and they asked if $n^{1/k}$ gives the correct order of magnitude of $c(n,k)$ for $k \geq 2$. Our main theorem answers this question as we prove that for every integer $k \geq 2$, and for sufficiently large $n$, $c(n,k) \leq k\lceil n^{1/k} \rceil + k^2$. This upper bound, together with the lower bound of Affif Chaouche et al., shows that the order of magnitude of $c(n,k)$ is $n^{1/k}$.

Keywords: pancyclicity, chords.

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References


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