

## KALEIDOSCOPIIC EDGE-COLORING OF COMPLETE GRAPHS AND $r$ -REGULAR GRAPHS<sup>1</sup>

XUELIANG LI AND XIAOYU ZHU

*Center for Combinatorics and LPMC  
Nankai University, Tianjin 300071, China*

**e-mail:** lxl@nankai.edu.cn  
zhuxy@mail.nankai.edu.cn

### Abstract

For an  $r$ -regular graph  $G$ , we define an edge-coloring  $c$  with colors from  $\{1, 2, \dots, k\}$ , in such a way that any vertex of  $G$  is incident with at least one edge of each color. The multiset-color  $c_m(v)$  of a vertex  $v$  is defined as the ordered tuple  $(a_1, a_2, \dots, a_k)$ , where  $a_i$  ( $1 \leq i \leq k$ ) denotes the number of edges of color  $i$  which are incident with  $v$  in  $G$ . Then this edge-coloring  $c$  is called a  $k$ -kaleidoscopic coloring of  $G$  if every two distinct vertices in  $G$  have different multiset-colors and in this way the graph  $G$  is defined as a  $k$ -kaleidoscope. In this paper, we determine the integer  $k$  for a complete graph  $K_n$  to be a  $k$ -kaleidoscope, and hence solve a conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] that for any integers  $n$  and  $k$  with  $n \geq k + 3 \geq 6$ , the complete graph  $K_n$  is a  $k$ -kaleidoscope. Then, we construct an  $r$ -regular 3-kaleidoscope of order  $\binom{r-1}{2} - 1$  for each integer  $r \geq 7$ , where  $r \equiv 3 \pmod{4}$ , which solves another conjecture in [P. Zhang, A Kaleidoscopic View of Graph Colorings, (Springer Briefs in Math., New York, 2016)] on the maximum order of  $r$ -regular 3-kaleidoscopes.

**Keywords:**  $k$ -kaleidoscope, regular graph, edge-coloring.

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