Discussiones Mathematicae
Graph Theory 39 (2019) 889–897
doi:10.7151/dmgt.2047

Full PDF

DMGT Page

FACIAL RAINBOW COLORING OF PLANE GRAPHS

Stanislav Jendroľ and Lucia Kekeňáková

Institute of Mathematics, Faculty of Science
Šafárik University, Jesenná 5
040 01 Košice, Slovakia

e-mail: stanislav.jendrol@upjs.sk
lucka.kekenakova@gmail.com

Abstract

A vertex coloring of a plane graph G is a *facial rainbow coloring* if any two vertices of G connected by a facial path have distinct colors. The *facial rainbow number* of a plane graph G, denoted by $rb(G)$, is the minimum number of colors that are necessary in any facial rainbow coloring of G. Let $L(G)$ denote the order of a longest facial path in G. In the present note we prove that $rb(T) \leq \left\lfloor \frac{3}{2}L(T) \right\rfloor$ for any tree T and $rb(G) \leq \left\lceil \frac{5}{3}L(G) \right\rceil$ for arbitrary simple graph G. The upper bound for trees is tight. For any simple 3-connected plane graph G we have $rb(G) \leq L(G) + 5$.

Keywords: cyclic coloring, rainbow coloring, plane graphs.

2010 Mathematics Subject Classification: 05C10, 05C15.

This work was supported by the Slovak VEGA Grant 1/0368/16 and by the Slovak Research and Development Agency under the contract no. APVV-15-0116.

References

[1] O. Amini, L. Esperet and J. van den Heuvel, *A unified approach to distance-two colouring of planar graphs*, in: Proc. SODA 2009 273–282. doi:10.1137/1.9781611973068.31

[2] K. Appel and W. Haken, *Every planar graph is four colorable, Part* I*: Discharging*, Illinois J. Math. **21** (1977) 429–490.

[3] K. Appel and W. Haken, *Every planar graph is four colorable, Part* II*: Reducibility*, Illinois J. Math. **21** (1977) 491–567.

[4] J. Azarija, R. Erman, D. Kráľ, M. Krnc and L. Stacho, *Cyclic colorings of plane graphs with independent faces*, European J. Combin. **33** (2012) 294–301. doi:10.1016/j.ejc.2011.09.011

[5] O.V. Borodin, *Solution of Ringel's problems on vertex-face coloring of plane graphs and coloring of* 1*-planar graphs*, Met. Diskret. Anal. **41** (1984) 12–26.

[6] O.V. Borodin, *Cyclic coloring of plane graphs*, Discrete Math. **100** (1992) 281–289. doi:10.1016/0012-365X(92)90647-X

[7] O.V. Borodin, *A new proof of the* 6 *color theorem*, J. Graph Theory **19** (1995) 507–521.
doi:10.1002/jgt.3190190406

[8] O.V. Borodin, *Colorings of plane graphs: A survey*, Discrete Math. **313** (2013) 517–539.
doi:10.1016/j.disc.2012.11.011

[9] O.V. Borodin, D.P. Sanders and Y. Zhao, *On cyclic colorings and their generalizations*, Discrete Math. **203** (1999) 23–40.
doi:10.1016/S0012-365X(99)00018-7

[10] J. Czap and S. Jendroľ, *Facially-constrained colorings of plane graphs: A survey*, Discrete Math. **340** (2017) 2691–2730.
doi:10.1016/j.disc.2016.07.026

[11] Z. Dvořák, M. Hebdige, F. Hlásek and D. Kráľ, *Cyclic coloring of plane graphs with maximum face size* 16 *and* 17*.*
arXiv:1603.06722v2[math.CO]

[12] Z. Dvořák, D. Kráľ and R. Škrekovski, *Non-rainbow coloring of* 3-*,* 4-*, and* 5-*connected plane graphs*, J. Graph Theory **63** (2010) 129–145.
doi:10.1002/jgt.20414

[13] H. Enomoto and M. Horňák, *A general upper bound for the cyclic chromatic number of* 3*-connected plane graphs*, J. Graph Theory **62** (2009) 1–25.
doi:10.1002/jgt.20383

[14] H. Enomoto, M. Horňák and S. Jendroľ, *Cyclic chromatic number of* 3*-connected plane graphs*, SIAM J. Discrete Math. **14** (2001) 121–137.
doi:10.1137/S0895480198346150

[15] F. Fujie-Okamoto, K. Kolasinski, J.W. Lin and P. Zhang, *Vertex rainbow colorings of graphs*, Discuss. Math. Graph Theory **32** (2012) 63–68.
doi:10.7151/dmgt.1586

[16] J.L. Gross and T.W. Tucker, Topological Graph Theory (Dover Publications, 2001).

[17] F. Havet, J.-S. Sereni and R. Škrekovski, 3*-facial coloring of plane graphs*, SIAM J. Discrete Math **22** (2008) 231–247.
doi:10.1137/060664124

[18] M. Hebdige and D. Kráľ, *Third case of the cyclic coloring conjecture.*
arxiv.org/pdf/1501.06624.pdf

[19] M. Horňák and S. Jendroľ, *On a conjecture by Plummer and Toft*, J. Graph Theory **30** (1999) 177–189.
doi:10.1002/(SICI)1097-0118(199903)30:3<177::AID-JGT3>3.0.CO;2-K

[20] M. Horňák and J. Zlámalová, *Another step towards proving a conjecture by Plummer and Toft*, Discrete Math. **310** (2010) 442–452.
doi:10.1016/j.disc.2009.03.016

[21] S. Jendroľ and L. Kekeňáková, *Facial rainbow colorings of trees*, Australas. J. Combin. **69** (2017) 358–367.

[22] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley, New York, 1995).

[23] Z. Jin, Y. Sun and J. Tu, *Rainbow total-coloring of complementary graphs and Erdős-Gallai type problem for the rainbow total connection number*, Discuss. Math. Graph Theory **38** (2018) 1023–1036.
doi:10.7151/dmgt.2056

[24] X. Li, Y. Shi and Y. Sun, *Rainbow connection of a graph; a survey*, Graphs Combin. **29** (2013) 1–38.
doi:10.1007/s00373-012-1243-2

[25] O. Ore and M.D. Plummer, *Cyclic coloration of plane graphs*, in: W.T. Tutte (Ed.), Recent Progress in Combinatorics (Proceedings of the Third Waterloo Conference on Combinatorics, May 1968) (Academic Press, 1969).

[26] M.D. Plummer and B. Toft, *Cyclic coloration of* 3*-polytopes*, J. Graph Theory **11** (1987) 507–515.
doi:10.1002/jgt.3190110407

[27] G. Ringel, *Ein Sechsfarbenproblem auf der Kugel*, Abh. Math. Semin. Univ. Hambg. **29** (1965) 107–117.
doi:10.1007/BF02996313

[28] N. Robertson, D.P. Sanders, P.D. Seymour and R. Thomas, *The four-colour theorem*, J. Combin. Theory Ser. B **70** (1999) 2–44.
doi:10.1006/jctb.1997.1750

[29] D.P. Sanders and Y. Zhao, *A new bound on the cyclic chromatic number*, J. Combin. Theory Ser. B **83** (2001) 102–111.
doi:10.1006/jctb.2001.2046

[30] D. West, Introduction to Graph Theory, Second Edition (Prentice Hall, Upper Saddle River, 2001).

Received 28 March 2017
Revised 19 December 2017
Accepted 19 December 2017