

THE PATH-PAIRABILITY NUMBER OF PRODUCT OF STARS

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Abstract

The study of a graph theory model of certain telecommunications network problems lead to the concept of path-pairability, a variation of weak linkedness of graphs. A graph G is k -path-pairable if for any set of $2k$ distinct vertices, s_i, t_i , $1 \leq i \leq k$, there exist pairwise edge-disjoint s_i, t_i -paths in G , for $1 \leq i \leq k$. The *path-pairability number* is the largest k such that G is k -path-pairable. Cliques, stars, the Cartesian product of two cliques (of order at least three) are ‘fully pairable’; that is $\lfloor n/2 \rfloor$ -pairable, where n is the order of the graph. Here we determine the path-pairability number of the Cartesian product of two stars.

Keywords: path-pairability, weak linkage, Cartesian product, star-like network, telecommunications network.

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