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HAMILTONIAN NORMAL CAYLEY GRAPHS

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Abstract

A variant of the Lovász Conjecture on hamiltonian paths states that *every finite connected Cayley graph contains a hamiltonian cycle.* Given a finite group G and a connection set S, the Cayley graph $Cay(G, S)$ will be called *normal* if for every $g \in G$ we have that $g^{-1}Sg = S$. In this paper we present some conditions on the connection set of a normal Cayley graph which imply the existence of a hamiltonian cycle in the graph.

Keywords: Cayley graph, hamiltonian cycle, normal connection set.

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