

HAMILTONICITIES OF DOUBLE DOMINATION CRITICAL AND STABLE CLAW-FREE GRAPHS

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Abstract

A graph G with the double domination number $\gamma_{\times 2}(G) = k$ is said to be k - $\gamma_{\times 2}$ -critical if $\gamma_{\times 2}(G + uv) < k$ for any $uv \notin E(G)$. On the other hand, a graph G with $\gamma_{\times 2}(G) = k$ is said to be k - $\gamma_{\times 2}^+$ -stable if $\gamma_{\times 2}(G + uv) = k$ for any $uv \notin E(G)$ and is said to be k - $\gamma_{\times 2}^-$ -stable if $\gamma_{\times 2}(G - uv) = k$ for any $uv \in E(G)$. The problem of interest is to determine whether or not 2-connected k - $\gamma_{\times 2}$ -critical graphs are Hamiltonian. In this paper, for $k \geq 4$, we provide a 2-connected k - $\gamma_{\times 2}$ -critical graph which is non-Hamiltonian. We prove that all 2-connected k - $\gamma_{\times 2}$ -critical claw-free graphs are Hamiltonian when $2 \leq k \leq 5$. We show that the condition claw-free when $k = 4$ is best possible. We further show that every 3-connected k - $\gamma_{\times 2}$ -critical claw-free graph is Hamiltonian when $2 \leq k \leq 7$. We also investigate Hamiltonian properties of k - $\gamma_{\times 2}^+$ -stable graphs and k - $\gamma_{\times 2}^-$ -stable graphs.

Keywords: double domination, critical, stable, Hamiltonian.

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REFERENCES

- [1] S. Ao, G. MacGillivray and J. Simmons, *Hamiltonian properties of independent domination critical graphs*, J. Combin. Math. Combin. Comput. **85** (2013) 107–128.
- [2] J. Brousek, Z. Ryjáček and O. Favaron, *Forbidden subgraphs, Hamiltonicity and closure in claw-free graphs*, Discrete Math. **196** (1999) 29–50.
doi:10.1016/S0012-365X(98)00334-3
- [3] V. Chvátal, *Tough graphs and Hamiltonian circuits*, Discrete Math. **306** (2006) 910–917 (reprinted from Discrete Math. **5** (1973) 215–228).
doi:10.1016/j.disc.2006.03.011

- [4] M. Chellali and T.W. Haynes, *Double domination stable graphs upon edge removal*, Australas. J. Combin. **47** (2010) 157–164.
- [5] O. Favaron, F. Tian and L. Zhang, *Independence and Hamiltonicity in 3-dominion critical graphs*, J. Graph Theory **25** (1997) 173–184.
doi:10.1002/(SICI)1097-0118(199707)25:3<173::AID-JGT1>3.0.CO;2-I
- [6] E. Flandrin, F. Tian, B. Wei and L. Zhang, *Some properties of 3-domination critical graphs*, Discrete Math. **205** (1999) 65–76.
doi:10.1016/S0012-365X(99)00038-2
- [7] Z. Ryjáček, *On a closure concept in claw-free graphs*, J. Combin. Theory Ser. B **70** (1997) 217–224.
doi:10.1006/jctb.1996.1732
- [8] P. Kaemawichanurat and L. Caccetta, *Hamiltonicity of domination critical claw-free graphs*, J. Combin. Math. Combin. Comput. **103** (2017) 39–62.
- [9] P. Kaemawichanurat, L. Caccetta and W. Ananchuen, *Hamiltonicities of connected domination critical graphs*, Ars Combin. **136** (2018) 137–151.
- [10] J. Simmons, *Closure Operations and Hamiltonian Properties of Independent and Total Domination Critical Graphs*, Ph.D. Thesis (University of Victoria, 2005).
- [11] D.P. Sumner and P. Blitch, *Domination critical graphs*, J. Combin. Theory Ser. B **34** (1983) 65–76.
doi:10.1016/0095-8956(83)90007-2
- [12] D.W. Thacker, *Double Domination Edge Critical Graph*, Master Thesis (East Tennessee State University, 2006).
- [13] F. Tian, B. Wei and L. Zhang, *Hamiltonicity in 3-domination critical graphs with $\alpha = \delta + 2$* , Discrete Appl. Math. **92** (1999) 57–70.
doi:10.1016/S0166-218X(98)00149-8
- [14] H.C. Wang and L.Y. Kang, *Matching properties in double domination edge critical graphs*, Discrete Math. Algorithms Appl. **2** (2010) 151–160.
doi:10.1142/S1793830910000541
- [15] H.C. Wang and E.F. Shan, *Some matching properties in $4-\gamma_{\times 2}$ -critical graphs*, Comput. Math. Appl. **59** (2010) 694–699.
doi:10.1016/j.camwa.2009.10.024
- [16] H.C. Wang, E.F. Shan and Y.C. Zhao, *3-factor criticality in double domination edge critical graphs*, Graphs Combin. **32** (2016) 1599–1610.
doi:10.1007/s00373-015-1670-y
- [17] E. Wojcicka, *Hamiltonian properties of domination critical graphs*, J. Graph Theory **14** (1990) 205–215.
doi:10.1002/jgt.3190140209
- [18] W. Xiong, H.J. Lai, X. Ma, K. Wang and M. Zhang, *Hamilton cycles in 3-connected claw-free and net-free graphs*, Discrete Math. **313** (2013) 784–795.
doi:10.1016/j.disc.2012.12.016

- [19] Y. Yuansheng, Z. Chengye, L. Xiaohui, J. Yongsong and H. Xin, *Some 3-connected 4-edge critical non-Hamiltonian graphs*, *J. Graph Theory* **50** (2005) 316–320.
doi:10.1002/jgt.20114

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