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A NOTE ON LOWER BOUNDS FOR INDUCED RAMSEY NUMBERS

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Abstract

We say that a graph F *strongly arrows* a pair of graphs (G,H) and write $F \overset{\text{ind}}{\longrightarrow} (G,H)$ if any 2-coloring of its edges with red and blue leads to either a red G or a blue H appearing as induced subgraphs of F. *The induced Ramsey number*, $IR(G,H)$ is defined as $\min\{|V(F)| : F \overset{\text{ind}}{\longrightarrow} (G,H)\}$. We will consider two aspects of induced Ramsey numbers. Firstly we will show that the lower bound of the induced Ramsey number for a connected graph G with independence number α and a graph H with clique number ω is roughly $\frac{\omega^2\alpha}{2}$. This bound is sharp. Moreover we will also consider the case when G is not connected providing also a sharp lower bound which is linear in both parameters.

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