TOTAL FORCING SETS AND ZERO FORCING SETS IN TREES

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Abstract \\

A dynamic coloring of the vertices of a graph $G$ starts with an initial subset $S$ of colored vertices, with all remaining vertices being non-colored. At each discrete time interval, a colored vertex with exactly one non-colored neighbor forces this non-colored neighbor to be colored. The initial set $S$ is called a forcing set of $G$ if, by iteratively applying the forcing process, every vertex in $G$ becomes colored. If the initial set $S$ has the added property that it induces a subgraph of $G$ without isolated vertices, then $S$ is called a total forcing set in $G$. The minimum cardinality of a total forcing set in $G$ is its total forcing number, denoted $F_t(G)$. We prove that if $T$ is a tree of order $n \geq 3$ with maximum degree $\Delta$ and with $n_1$ leaves, then $n_1 \leq F_t(T) \leq \frac{1}{\Delta}((\Delta - 1)n + 1)$. In both lower and upper bounds, we characterize the infinite family of trees achieving equality. Further we show that $F_t(T) \geq F(T) + 1$, and we characterize the extremal trees for which equality holds.

Keywords: forcing set, forcing number, total forcing set, total forcing number.

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References


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