ON FACTORABLE BIGRAPHIC PAIRS

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Abstract

Let $S = (a_1, \ldots, a_m; b_1, \ldots, b_n)$, where $a_1, \ldots, a_m$ and $b_1, \ldots, b_n$ are two sequences of nonnegative integers. We say that $S$ is a bigraphic pair if there exists a simple bipartite graph $G$ with partite sets $\{x_1, x_2, \ldots, x_m\}$ and $\{y_1, y_2, \ldots, y_n\}$ such that $d_G(x_i) = a_i$ for $1 \leq i \leq m$ and $d_G(y_j) = b_j$ for $1 \leq j \leq n$. In this case, we say that $G$ is a realization of $S$. Analogous to Kundu’s $k$-factor theorem, we show that if $(a_1, a_2, \ldots, a_m; b_1, b_2, \ldots, b_n)$ and $(a_1 - e_1, a_2 - e_2, \ldots, a_m - e_m; b_1 - f_1, b_2 - f_2, \ldots, b_n - f_n)$ are two bigraphic pairs satisfying $k \leq f_i \leq k + 1$, $1 \leq i \leq n$ (or $k \leq e_i \leq k + 1$, $1 \leq i \leq m$), for some $0 \leq k \leq m - 1$ (or $0 \leq k \leq n - 1$), then $(a_1, a_2, \ldots, a_m; b_1, b_2, \ldots, b_n)$ has a realization containing an $(e_1, e_2, \ldots, e_m; f_1, f_2, \ldots, f_n)$-factor. For $m = n$, we also give a necessary and sufficient condition for an $(k^n; k^n)$-factorable bigraphic pair to be connected $(k^n; k^n)$-factorable when $k \geq 2$. This implies a characterization of bigraphic pairs with a realization containing a Hamiltonian cycle.

Keywords: degree sequence, bigraphic pair, Hamiltonian cycle.

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References


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