CORRIGENDUM TO: INDEPENDENT TRANSVERSAL DOMINATION IN GRAPHS [DISCUSS. MATH. GRAPH THEORY 32 (2012) 5–17]

EMMA GUZMAN-GARCIA

AND

ROCÍO SÁNCHEZ-LÓPEZ

Facultad de Ciencias, Universidad Nacional Autónoma de México
Circuito Exterior s/n, Coyocacán,
Ciudad Universitaria, 04510
Ciudad de México, CDMX

e-mail: guzmanemma.604@gmail.com
usagitsukinomx@yahoo.com.mx

Abstract

In [Independent transversal domination in graphs, Discuss. Math. Graph Theory 32 (2012) 5–17], Hamid claims that if $G$ is a connected bipartite graph with bipartition $\{X,Y\}$ such that $|X| \leq |Y|$ and $|X| = \gamma(G)$, then $\gamma_{it}(G) = \gamma(G) + 1$ if and only if every vertex $x$ in $X$ is adjacent to at least two pendant vertices. In this corrigendum, we give a counterexample for the sufficient condition of this sentence and we provide a right characterization.

On the other hand, we show an example that disproves a construction which is given in the same paper.

Keywords: domination, independent, transversal, covering, matching.

2010 Mathematics Subject Classification: 05C69.

1. Introduction

Among the results that Hamid shows in [4] we find the following.

**Theorem 1.1** [4]. Let $G$ be a connected bipartite graph with bipartition $\{X,Y\}$ such that $|X| \leq |Y|$ and $|X| = \gamma(G)$. Then $\gamma_{it}(G) = \gamma(G) + 1$ if and only if every vertex $x$ in $X$ is adjacent to at least two pendant vertices.
We find a connected bipartite graph $G$ with bipartition $\{X,Y\}$ such that $|X| \leq |Y|$, $|X| = \gamma(G)$ and $\gamma_{\mu}(G) = \gamma(G) + 1$. But there exists a vertex in $X$ which is not adjacent to at least two pendant vertices.

A problem that arises with Theorem 1.1 is that it is used in [1] in order to prove the following result.

**Corollary 1.2** [1]. Let $T$ be a tree with bipartition $\{X,Y\}$ such that $1 \leq |X| \leq |Y|$ and $\gamma(T) = |X|$. Then, $\gamma_{\mu}(T) = \gamma(T)$ if and only if there is a vertex in $X$ which is adjacent to at most one pendant vertex.

In this corrigendum, we provide a right characterization for bipartite graphs $G$ with bipartition $\{X,Y\}$, $|X| \leq |Y|$ and $|X| = \gamma(G)$, such that $\gamma_{\mu}(G) = \gamma(G) + 1$. As a consequence of the main result, we show the corrected version of Corollary 1.2.

Other result showed in [4] is the following.

**Theorem 1.3** [4]. Let $a$ and $b$ be two positive integers with $b \geq 2a - 1$. Then there exists a connected graph $G$ on $b$ vertices such that $\gamma_{\mu}(G) = a$.

In order to prove Theorem 1.3, Hamid proposes the following construction: set $b = 2a + r$, with $r \geq -1$, and let $H$ be any connected graph on $a$ vertices. Let $V(H) = \{v_1, v_2, \ldots, v_a\}$ be the vertex set of $H$ and let $G$ be the graph obtained from $H$ by attaching $r+1$ pendant edges at $v_1$ and one pendant edge at each $v_i$, for $i \geq 2$. Let $u_i$ be the pendant vertex in $G$ adjacent to $v_i$, for $i \geq 2$.

Hamid claims that $\gamma_{\mu}(G) = a$ and $S = \{v_1, u_2, u_3, \ldots, u_a\}$ is a $\gamma_{\mu}(G)$-set. Further, every maximum independent set of $G$ intersects $S$ and hence $\gamma_{\mu}(G) = a$.

We find that, in some cases for $H$, $G$ holds $\gamma_{\mu}(G) \neq a$ and there exists an $\alpha(G)$-set which does not intersect $S$.

In this corrigendum we provide a correct proof of Theorem 1.3 for $b \geq 2a$.

2. **Definitions and Known Results**

We use [2] and [3] for terminology and notation not defined here and consider finite and simple graphs only. For introductory notation, let $G$ be a graph. $n(G)$ denotes $|V(G)|$. Let $v$ be a vertex of $G$, the open neighborhood of $v$ in $G$, denoted by $N(v)$, is defined as the set $\{u \in V(G): uv \in E(G)\}$. The degree of a vertex $v$, denoted by $\delta(v)$, is the number $|N(v)|$. We say that a vertex $u$ is a pendant vertex if $\delta(u) = 1$. For a graph $G$, the number $\min\{\delta(u): u \in V(G)\}$ is denoted by $\delta(G)$. An edge of a graph is said to be a pendant edge if one of its vertices is a pendant vertex. A complete graph is a graph with $n$ vertices and an edge between every two vertices, denoted by $K_n$. A subset $I$ of $V(G)$ is said to be independent if every two vertices of $I$ are non-adjacent. We say that a graph $G$ is bipartite if
there exists a partition \( \{X, Y\} \) of \( V(G) \) such that \( X \) and \( Y \) are independent sets (we call that partition a bipartition). If \( G \) contains every edge joining \( X \) and \( Y \), then \( G \) is a complete bipartite graph, denoted by \( K_{m,n} \) with \( |X| = m \) and \( |Y| = n \).

The complete bipartite graph \( K_{1,n} \) is called a star.

A subset \( D \) of \( V(G) \) is said to be dominating if for every \( u \) in \( V(G) - D \) it holds \( N(u) \cap D \neq \emptyset \). The cardinality of a smallest dominating set is the domination number, denoted by \( \gamma(G) \), and we refer to such a set as a \( \gamma(G) \)-set.

The cardinality of a largest independent set in \( G \) is the independence number, denoted by \( \alpha(G) \), and an independent set having cardinality \( \alpha(G) \) is called a maximum independent set. We refer to such a set as an \( \alpha(G) \)-set. A subset \( M \) of \( E(G) \) is a matching if every two edges of \( M \) are non-adjacent. A maximum matching is one of largest cardinality in \( G \). The number of edges in a maximum matching of a graph \( G \) is called the matching number of \( G \), denoted by \( \beta(G) \). A subset \( K \) of \( V(G) \) such that every edge of \( G \) has at least one end in \( K \) is called a covering of \( G \). The number of vertices in a minimum covering of \( G \) is the covering number of \( G \), denoted by \( \delta(G) \). An independent transversal dominating set in \( G \) is a dominating set that intersects every maximum independent set in \( G \). The independent transversal domination number, denoted by \( \gamma_{it}(G) \), is the smallest cardinality of an independent transversal dominating set of \( G \). An independent transversal dominating set of cardinality \( \gamma_{it}(G) \) is called a minimum independent transversal dominating set. We refer to such a set as a \( \gamma_{it}(G) \)-set.

We need the following results.

**Theorem 2.1** [5]. For any tree \( T \), \( \gamma(T) = n(T) - \Delta(T) \) if and only if \( T \) is a wounded spider.

**Proposition 2.1** ([4], Example 3.1). \( \gamma_{it}(K_{m,n}) = 2 \).

**Theorem 2.2** [4]. For any graph \( G \), we have \( \gamma(G) \leq \gamma_{it}(G) \leq \gamma(G) + \delta(G) \).

**Lemma 2.3** ([2], page 74). Let \( M \) be a matching and \( K \) a covering such that \( |M| = |K| \). Then \( M \) is a maximum matching and \( K \) is a minimum covering.

**Lemma 2.4** ([2], page 101). Let \( G \) be a graph. Then \( \alpha(G) + \beta(G) = n(G) \).

### 3. A Counterexample for Theorem 1.1

Consider the graph \( G \) in Figure 1. Since \( M = \{x_1y_2, x_2y_5, x_3y_4, x_4y_6\} \) is a matching and \( X \) is a covering such that \( |M| = |X| \), it follows from Lemmas 2.3 and 2.4 that \( \alpha(G) = 7 \); also it is straightforward to see that \( \gamma(G) = 4 \). On the other hand, notice that \( X \) and \( (X - \{x_4\}) \cup \{y_6\} \) are the only one \( \gamma(G) \)-sets. Therefore, since \( Y \) and \( (Y - \{y_6\}) \cup \{x_4\} \) are \( \alpha(G) \)-sets such that \( X \cap Y = \emptyset \) and \( (Y - \{y_6\}) \cup \{x_4\} \)
∩ ((X – {x₄}) ∪ {y₆}) = ∅, we get from Theorem 2.2 that γ_{it}(G) = γ(G) + 1 (because \( \delta(G) = 1 \)). As \( x₄ \) is not adjacent to at least two pendant vertices, we obtain a counterexample for Theorem 1.1.

Figure 1. \( N(x₄) \) has no pendant vertices.

4. RIGHT CHARACTERIZATION FOR BIPARTITE GRAPHS \( G \) SUCH THAT
   \[ |X| \leq |Y|, |X| = \gamma(G) \text{ AND } \gamma_{it}(G) = \gamma(G) + 1 \]

We need the following results.

**Corollary 4.1** [4]. If \( G \) has an isolated vertex, then \( \gamma_{it}(G) = \gamma(G) \).

Theorem 4.2 shows the right version of Theorem 1.1. Moreover, Theorem 4.2 allows disconnected graphs.

**Theorem 4.2.** Let \( G \) be a bipartite graph with bipartition \( \{X, Y\} \) such that \( |X| \leq |Y| \) and \( |X| = \gamma(G) \). Then \( \gamma_{it}(G) = \gamma(G) + 1 \) if and only if

1. every vertex \( x \) in \( X \), such that \( \delta(x) \neq 1 \), is adjacent to at least two pendant vertices,
2. \( Y \) has no isolated vertices.

**Proof.** If \( |V(G)| = 2 \), hypothesis \( |X| = \gamma(G) \) implies that \( G = K₂ \) and therefore \( G \) satisfies Theorem 4.2. Assume that \( |V(G)| \geq 3 \).

Suppose that \( \gamma_{it}(G) = \gamma(G) + 1 \). It follows from Corollary 4.1 that \( G \) has no isolated vertices, which implies that \( \delta(G) \geq 1 \). Therefore, in particular \( Y \) has no isolated vertices. Thus, it remains to prove that every vertex \( x \) in \( X \), such that \( \delta(x) \neq 1 \), is adjacent to at least two pendant vertices. Suppose that there exists a vertex \( w \) in \( X \) such that \( \delta(w) \geq 2 \).

Notice that \( X \) is a \( \gamma(G) \)-set (because for every \( u \) in \( (V(G) - X) = Y \), \( \delta(u) \geq 1 \), and \( |X| = \gamma(G) \)).
Consider the following claims.

Claim 1. $\alpha(G) = |Y|$.

Given that $Y$ is an independent set in $G$, we get that $\alpha(G) \geq |Y|$. On the other hand, the hypotheses $\gamma_{it}(G) = \gamma(G) + 1$ and $|X| = \gamma(G)$ imply that there exists an $\alpha(G)$-set $S$ such that $X \cap S = \emptyset$. Since $S \subseteq Y$, then $\alpha(G) = |S| \leq |Y|$. Therefore, $\alpha(G) = |Y|$.

Claim 2. $\delta(G) = 1$.

Proceeding by contradiction, suppose that $\delta(G) \geq 2$. Let $u$ and $v$ be two vertices in $G$ such that $u \in X$ and $v \in N(u)$. Set $S = (X - \{u\}) \cup \{v\}$.

Claim 2.1. $S$ is a dominating set in $G$.

Since $\delta(w) \geq 2$ for every $w$ in $Y - \{v\}$, there exists $x_w$ in $X - \{u\}$ such that $wx_w \in E(G)$. Therefore, $S$ is a dominating set in $G$ (consider the choice of $v$).

Claim 2.2. $S \cap J \neq \emptyset$ for every $\alpha(G)$-set $J$.

Let $J$ be an $\alpha(G)$-set. If $v \in J$, then $S \cap J \neq \emptyset$. Suppose that $v \notin J$. Given that $|J| = \alpha(G) = |Y|$ (by Claim 1) and $v \notin J$, it follows that $X \cap J \neq \emptyset$. If $u \notin J$, we get $(X - \{u\}) \cap J \neq \emptyset$ (because $X \cap J \neq \emptyset$), which implies that $S \cap J \neq \emptyset$. Thus, suppose that $u \in J$. Since $\delta(u) \geq 2$, there exists $z$ in $Y - \{v\}$ such that $uz \in E(G)$, which implies that $|J \cap Y| \leq |Y| - 2$ (because $u \in J$, $\{uw, uz\} \subseteq E(G)$ and $J$ is an independent set). Therefore, $2 \leq |X \cap J|$, which implies that $(X - \{u\}) \cap J \neq \emptyset$. Thus, $S \cap J \neq \emptyset$.

We get from Claims 2.1, 2.2, the definition of $S$ and the hypothesis that $\gamma_{it}(G) \leq |S| = |X| = \gamma(G)$, a contradiction with $\gamma_{it}(G) = \gamma(G) + 1$. Therefore, $\delta(G) = 1$.

Let $u$ be a vertex in $X$ such that $\delta(u) \geq 2$. We will prove that $u$ is adjacent to at least two pendant vertices. Proceeding by contradiction, suppose that $N(u)$ contains at most one pendant vertex. If $N(u)$ contains a pendant vertex $v$, choose $v$, otherwise let $v$ be any vertex in $N(u)$. Set $S = (X - \{u\}) \cup \{v\}$.

Claim 3. $S$ is a dominating set in $G$.

Given that $\delta(w) \geq 1$ for every $w$ in $Y - N(u)$, it follows that there exists $x_w$ in $X - \{u\}$ such that $wx_w \in E(G)$. On the other hand, since for every $z$ in $N(u) - \{v\}$ it holds that $\delta(z) \geq 2$, then there exists $x_z$ in $X - \{u\}$ such that $zx_z \in E(G)$. Therefore, $S$ is a dominating set in $G$.

Claim 4. If $J$ is an $\alpha(G)$-set, then $S \cap J \neq \emptyset$.

The proof is the same as the proof of Claim 2.2.
We get from Claims 3, 4, the definition of $S$ and the hypothesis that $\gamma_{it}(G) \leq |S| = |X| = \gamma(G)$, a contradiction with $\gamma_{it}(G) = \gamma(G) + 1$. Hence, $u$ is adjacent to at least two pendant vertices.

Therefore, every vertex $x$ in $X$, such that $\delta(x) \neq 1$, is adjacent to at least two pendant vertices.

Suppose that for every vertex $w$ in $X$, such that $\delta(w) \neq 1$, $N(w)$ contains at least two pendant vertices and $Y$ has no isolated vertices. Notice that it follows from the hypothesis that $\delta(G) \geq 1$. Consider the following claims.

**Claim A.** $\alpha(G) = |Y|$.

Given that $Y$ is an independent set, we get that $\alpha(G) \geq |Y|$. Proceeding by contradiction, suppose that $\alpha(G) > |Y|$ and let $J$ be an $\alpha(G)$-set.

Since $\alpha(G) > |Y|$ and $|X| \leq |Y|$, we get that $J \cap X \neq \emptyset$ and $J \cap Y \neq \emptyset$. Set $X' = J \cap X$, $Y' = J \cap Y$, $X_1 = \{x \in X' : \delta(x) \geq 2\}$ and $X_2 = \{x \in X' : \delta(x) = 1\}$.

**Claim A.1.** $|X_1| \geq 1$.

As $|Y| = |Y'| + |Y' - Y'|$, $|J| = |X'| + |Y'|$ and $|J| > |Y|$, it follows that $|X'| > |Y' - Y'|$, which implies that there exist two vertices in $X'$, say $u_1$ and $u_2$, and there exists a vertex $y$ in $Y' - Y'$ such that $\{u_1y, u_2y\} \subseteq E(G)$.

Proceeding by contradiction, suppose that $X_1 = \emptyset$. Since $\delta(u_1) = 1$ and $\delta(u_2) = 1$, then for every $z$ in $Y - (Y' \cup \{y\})$ there exists $x_z$ in $X - \{u_1, u_2\}$ such that $xz \in E(G)$ (recall that $\delta(G) \geq 1$). On the other hand, given that $J$ is an independent set, we get that for every $w$ in $Y'$ there exists $x_w$ in $X - X'$ such that $wx_w \in E(G)$. Hence, $(X - \{u_1, u_2\}) \cup \{y\}$ is a dominating set, a contradiction with $|X| = \gamma(G)$. Therefore, $|X_1| \geq 1$.

Since $N(X') \subseteq Y - Y'$ and every vertex of $X_1$ is adjacent to at least two pendant vertices, we get from the definition of $X_2$ that $|Y - Y'| \geq 2|X_1| + |X_2|$; that is, $|Y - Y'| \geq |X'| + |X_1|$, which implies that $|X_1| + |X'| + |Y'| \leq |Y|$. Hence, since $|X_1| + |J| \leq |Y|$, $1 \leq |X_1|$ (by Claim A.1) and $|Y| < |J|$, we get a contradiction.

Therefore, $\alpha(G) = |Y|$.

**Claim B.** If $D$ is a $\gamma(G)$-set, then $V(G) - D$ is an $\alpha(G)$-set.

Let $D$ be a $\gamma(G)$-set. Since $|D| = \gamma(G) = |X|$, then $|V(G) - D| = (|V(G)| - |X|) = |Y| = \alpha(G)$ (by Claim A). It remains to prove that $V(G) - D$ is an independent set. It is clear that $V(G) - D$ is an independent set if either $(V(G) - D) \subseteq X$ or $(V(G) - D) \subseteq Y$. Hence, suppose that $(V(G) - D) \cap X \neq \emptyset$ and $(V(G) - D) \cap Y \neq \emptyset$. Let $u$ and $v$ be two vertices in $V(G) - D$; we will prove that $uv \notin E(G)$. Suppose that $u \in (V(G) - D) \cap X$ and $v \in (V(G) - D) \cap Y$.

**Claim B.1.** $\delta(u) = 1$. 

Proceeding by contradiction, suppose that \( \delta(u) \geq 2 \). It follows from the hypothesis that \( N(u) \) has at least two pendant vertices, say \( w \) and \( z \). Since \( u \notin D \), we get that \( \{w, z\} \subseteq D \) (because \( D \) is a dominating set).

We will see that \( S = (D - \{w, z\}) \cup \{u\} \) is a dominating set. Notice that \( V(G) - S = (((V(G) - D) \cap X) - \{u\}) \cup (((V(G) - D) \cap Y) \cup \{w, z\}) \), \( D = (D \cap X) \cup (D \cap Y) \) and \( S = (D \cap X) \cup ((D \cap Y) - \{w, z\}) \cup \{u\} \). Given that \( D \) is a dominating set, we get that for every \( y \) in \( V(G) - D \cap Y \) there exists \( x_y \) in \( D \cap X \) such that \( yx_y \in E(G) \). In the same way for every \( x \) in \((V(G) - D) \cap X - \{u\}\) there exists \( y_x \) in \( D \cap Y \) such that \( xy_x \in E(G) \) (because \( w \) and \( z \) are pendant vertices which are adjacent to \( u \)). Hence, we conclude that \( S \) is a dominating set. Since \( |S| = |X| - 1 \), we get a contradiction with \( |X| = \gamma(G) \).

Therefore, \( \delta(u) = 1 \).

Given that \( \delta(u) = 1 \), \( u \notin D \) and \( D \) is a dominating set, it follows that \( N(u) \subseteq D \), which implies that \( uv \notin E(G) \) (because \( v \notin D \)).

Therefore, \( V(G) - D \) is an independent set. Hence, \( V(G) - D \) is an \( \alpha(G) \)-set.

Claim C. \( \delta(G) = 1 \).

Recall that \( \delta(G) \geq 1 \). If \( X \) has a pendant vertex, then we are done; otherwise, it follows from the hypothesis that for \( u \) in \( X \) there exists a pendant vertex in \( N(u) \). Therefore, \( \delta(G) = 1 \).

It follows from Claim B that \( \gamma_{it}(G) \neq \gamma(G) \). Therefore, we get from Claim C and Theorem 2.2 that \( \gamma_{it}(G) = \gamma(G) + 1 \).

5. Some Consequences of Theorem 4.2

A subdivision of an edge \( uv \) is obtained by replacing the edge \( uv \) with a path \( (u, w, v) \), where \( w \) is a new vertex. For a positive integer \( t \), a wounded spider is a star \( K_{1,t} \) with at most \( t - 1 \) of its edges subdivided. Similarly, for an integer \( t \geq 2 \), a healthy spider is a star \( K_{1,t} \) with all of its edges subdivided.

Remark 5.1. It is straightforward to see that if \( G \) is a healthy spider, then \( \gamma(G) = \Delta(G) \). On the other hand, if \( G \) is a healthy spider, it follows from Theorem 4.2 that \( \gamma_{it}(G) = \gamma(G) + 1 \).

Remark 5.2. Let \( G \) be a wounded spider which is not a star. Suppose that \( G \) is obtained from \( K_{1,t} \) by subdividing \( r \) of its edges, with \( 1 \leq r \leq t - 1 \) and \( t \geq 2 \).

1. If \( r \leq t - 2 \), then \( \gamma_{it}(G) = \gamma(G) + 1 = r + 2 \).
2. If \( r = t - 1 \), then \( \gamma_{it}(G) = \gamma(G) = t \).

Proof. Suppose that \( V(G) = \{u_1, v_2, \ldots, v_t, v_{t+1}\} \cup \{u_2, \ldots, u_r, u_{r+1}\} \), \( E(G) = \{u_1v_j : j \in \{2, \ldots, t + 1\}\} \cup \{u_iv_i : i \in \{2, \ldots, r + 1\}\} \). Set \( X = \{u_1, u_2, \ldots, u_r, u_{r+1}\} \) and \( Y = \{v_2, \ldots, v_t, v_{t+1}\} \).
1. Suppose that $r \leq t - 2$. It follows from Theorem 2.1 that $\gamma(G) = ((t+1)+r) - t = r + 1$ which implies that $|X| = \gamma(G)$. Therefore, we get from Theorem 4.2 that $\gamma_{it}(G) = \gamma(G) + 1 = (r+1)+1$.

2. Suppose that $r = t - 1$. It follows from Theorem 2.1 that $\gamma(G) = t$. Since $|X| = \gamma(G)$ and $u_1$ is not adjacent to at least two pendant vertices in $G$, it follows from Theorem 4.2 that $\gamma_{it}(G) \neq \gamma(G) + 1$. Therefore, given that $\delta(G) = 1$, we get from Theorem 2.2 that $\gamma_{it}(G) = \gamma(G)$. Hence, $\gamma_{it}(G) = t$.

**Corollary 5.1.** Let $T$ be a tree with bipartition $\{X,Y\}$ such that $1 \leq |X| \leq |Y|$ and $\gamma(T) = |X|$. Then, $\gamma_{it}(T) = \gamma(T)$ if and only if there is a vertex $x$ in $X$, with $\delta(x) \neq 1$, which is adjacent to at most one pendant vertex.

6. **Example Disproving Construction in Theorem 1.3**

Recall that, in order to prove Theorem 1.3, Hamid proposes the following construction: set $b = 2a + r$, with $r \geq -1$, and let $H$ be any connected graph on $a$ vertices. Let $V(H) = \{v_1, v_2, \ldots, v_a\}$ be the vertex set of $H$ and let $G$ be the graph obtained from $H$ by attaching $r+1$ pendant edges at $v_1$ and one pendant edge at each $v_i$, for $i \geq 2$. Let $u_i$ ($i \geq 2$) be the pendant vertex in $G$ adjacent to $v_i$.

Hamid claims that $\gamma_{it}(G) = a$ and $S = \{v_1, u_2, u_3, \ldots, u_a\}$ is a $\gamma_{it}(G)$-set. Further, every maximum independent set of $G$ intersects $S$ and hence $\gamma_{it}(G) = a$.

- We find that, when $r = -1$ and $a \geq 3$, for the graph $H = K_a$, the associated graph $G$ does not hold the conclusion of Theorem 1.3, see Figure 2.

In this case, since $K = (V(H) - \{v_1\})$ is a covering and $M = \{v_iu_i : i \in \{2, \ldots, a\}\}$ is a matching such that $|K| = a - 1 = |M|$, we get from Lemma 2.3 that $|K| = \beta(G)$. Thus, it follows from Lemma 2.4 that $\alpha(G) = 2a - 1 - (a - 1) = a$. Hence $(V(G) - K) = \{u_2, \ldots, u_a, v_1\}$ is the only one independent set in $G$ such that $|V(G) - K| = \alpha(G)$. Therefore, $V(G) - ((V(H) - \{v_a\}) \cup \{u_a\})$ is an
independent transversal dominating set in $G$, which implies that $\gamma_t(G) \leq a - 1$. On the other hand, let $S$ be a $\gamma_t(G)$-set. Given that $S$ is a dominating set, then $\{v_i, u_i\} \cap S \neq \emptyset$ for every $i$ in $\{2, \ldots, a\}$, which implies that $a - 1 \leq |S|$. Therefore, $\gamma_t(G) = a - 1$

- We find that, when $r > 0$ and $a \geq 2$, for the graph $H = K_{1,a-1}$, the associated graph $G$ is a wounded spider and this does not hold the conclusion of Theorem 1.3, see Figure 3.

\[ G : \]

![Figure 3](image)

Notice that $G$ is also obtained from $K_{1,a+r}$ by subdividing exactly $a - 1$ of its edges, where $a - 1 \leq (a + r) - 2$. Therefore, it follows from Remark 5.2 that $\gamma_t(G) = \gamma(G) + 1 = (a - 1) + 2 = a + 1$.

- When $r > 0$ and $a = 1$ we have that $G = K_{1,r+1}$ and in this case we get from Proposition 2.1 that $\gamma_t(G) = 2 = a + 1$, see Figure 4.

\[ G : \]

![Figure 4](image)

- When $H = K_{1,a-1}$, for $r \geq 0$ and $a \geq 2$, there exists an $\alpha(G)$-set in $G$ which does not intersect $S = \{v_1, v_2, u_3, \ldots, u_a\}$.

For every $i$ in $\{1, \ldots, r+1\}$ let $x_i$ be the pendant vertex adjacent to $v_1$. Since $M = \{v_1 x_1, v_2 x_2, \ldots, v_a x_a\}$ is a matching and $K = V(H)$ is a covering such that $|M| = |K|$, then we get from Lemma 2.3 that $K$ is a minimum covering. On the other hand, it follows from Lemma 2.4 that $2a + r = |V(G)| = \alpha(G) + \beta(G) = \alpha(G) + a$, which implies that $\alpha(G) = a + r$.

Therefore $(V(H) - \{v_1\}) \cup \{x_1, \ldots, x_{r+1}\}$ is an $\alpha(G)$-set in $G$ which does not intersect $S$. 
For $b \geq 2a$ we proceed to prove the following.

**Theorem 6.1.** Let $a$ and $b$ be two positive integers with $b \geq 2a$. Then there exists a connected graph $G$ on $b$ vertices such that $\gamma_{it}(G) = a$.

**Proof.** Suppose that $b = 2a + r$, for some $r \in \mathbb{N}$. Let $H$ be a connected graph of order $a$, such that $H \not\cong K_{1,a-1}$, with vertex set $V(H) = \{v_1, \ldots, v_a\}$. Let $\{x_1, \ldots, x_{r+1}\}$ and $\{u_2, \ldots, u_a\}$ be two sets such that $\{x_1, \ldots, x_{r+1}\} \cap \{u_2, \ldots, u_a\} = \emptyset$, $\{x_1, \ldots, x_{r+1}\} \cap V(H) = \emptyset$ and $V(H) \cap \{u_2, \ldots, u_a\} = \emptyset$. Let $G$ be the graph with $V(G) = V(H) \cup \{x_1, \ldots, x_{r+1}\} \cup \{u_2, \ldots, u_a\}$ and $E(G) = E(H) \cup \{v_iu_i : i \in \{2, \ldots, a\}\} \cup \{v_1x_i : i \in \{1, \ldots, r+1\}\}$.

**Claim 1.** $a \leq \gamma_{it}(G)$.

We will prove that $\gamma(G) = a$. Since $V(H)$ is a dominating set in $G$, then $\gamma(G) \leq a$. On the other hand, let $S$ be a $\gamma(G)$-set. Given that $\{u_i, v_i\} \cap S \neq \emptyset$ (because $S$ is a dominating set) for every $i$ in $\{2, \ldots, a\}$ and $r + 1 \geq 1$ we get that $|S| \geq a$. Hence, $\gamma(G) = a$. Therefore, it follows from Theorem 2.2 that $a \leq \gamma_{it}(G)$.

**Claim 2.** $\alpha(G) = r + a$.

Since $K = V(H)$ is a covering and $M = (\{v_iu_i : i \in \{2, \ldots, a\}\} \cup \{v_1x_1\})$ is a matching such that $|K| = a = |M|$, it follows from Lemma 2.3 that $|K| = \beta(G)$. Hence, we get from Lemma 2.4 that $\alpha(G) = r + a$.

**Claim 3.** $S = \{v_1, u_2, \ldots, u_a\}$ is an independent transversal dominating set in $G$.

Given that $S$ is a dominating set, it remains to prove that $S$ intersects every maximum independent set in $G$. Since $H \not\cong K_{1,a-1}$ and $H$ is connected, we get that $V(H) - \{v_1\}$ is not an independent set in $G$, which implies that $(V(H) - \{v_1\}) \cup \{x_1, \ldots, x_{r+1}\}$ is not an independent set in $G$. Since $|(V(H) - \{v_1\}) \cup \{x_1, \ldots, x_{r+1}\}| = a + r$, it follows that $S$ intersects every maximum independent set in $G$.

Therefore, we get from Claims 1 and 3 that $a \leq \gamma_{it}(G) \leq a$.

**Acknowledgements**

We thank the referees for their suggestions which improved the rewriting of this paper.

**References**

Corrigendum to: Independent Transversal Domination in Graphs


   doi:10.7151/dmgt.1581


Received 29 May 2019
Revised 30 December 2019
Accepted 3 January 2020