

GRAPHS WITH ALL BUT TWO EIGENVALUES IN $[-2, 0]$

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Abstract

The eigenvalues of a graph are those of its adjacency matrix. Recently, Cioabă, Haemers and Vermette characterized all graphs with all but two eigenvalues equal to -2 and 0 . In this article, we extend their result by characterizing explicitly all graphs with all but two eigenvalues in the interval $[-2, 0]$. Also, we determine among them those that are determined by their spectrum.

Keywords: graph spectrum, complete multipartite graph, graph determined by its spectrum.

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REFERENCES

- [1] A.E. Brouwer and W.H. Haemers, *Spectra of Graphs* (Springer, New York, 2012).
doi:10.1007/978-1-4614-1939-6
- [2] D. Cao and H. Yuan, *Graphs characterized by the second eigenvalue*, *J. Graph Theory* **17** (1993) 325–331.
doi:10.1002/jgt.3190170307
- [3] S.M. Cioabă, W.H. Haemers and J.R. Vermette, *The graphs with all but two eigenvalues equal to -2 or 0* , *Des. Codes Cryptogr.* **84** (2017) 153–163.
doi:10.1007/s10623-016-0241-4
- [4] S.M. Cioabă, W.H. Haemers, J.R. Vermette and W. Wong, *The graphs with all but two eigenvalues equal to ± 1* , *J. Algebraic Combin.* **41** (2015) 887–897.
doi:10.1007/s10801-014-0557-y
- [5] D. Cvetković, *On graphs whose second largest eigenvalue does not exceed 1*, *Publ. Inst. Math. (Beograd) (N.S.)* **31(45)** (1982) 15–20.
- [6] D. Cvetković, M. Doob and H. Sachs, *Spectra of Graphs: Theory and Applications*, 3rd Edition (Johann Ambrosius Barth Verlag, Heidelberg-Leipzig, 1995).
- [7] D. Cvetković and S. Simić, *On graphs whose second largest eigenvalue does not exceed $(\sqrt{5} - 1)/2$* , *Discrete Math.* **138** (1995) 213–227.
doi:10.1016/0012-365X(94)00204-V
- [8] D. Cvetković, M. Doob and H. Sachs, *Spectra of Graphs: Theory and Application* (Academic Press, New York, 1980).
- [9] L.S. de Lima, A. Mohammadian and C.S. Oliveira, *The non-bipartite graphs with all but two eigenvalues in $[-1, 1]$* , *Linear Multilinear Algebra* **65** (2017) 526–544.
doi:10.1080/03081087.2016.1194802
- [10] F. Esser and F. Harary, *On the spectrum of a complete multipartite graph*, *European J. Combin.* **1** (1980) 211–218.
doi:10.1016/S0195-6698(80)80004-7
- [11] H. Ma and H. Ren, *On the spectral characterization of the union of complete multipartite graph and some isolated vertices*, *Discrete Math.* **310** (2010) 3648–3652.
doi:10.1016/j.disc.2010.09.004
- [12] M. Petrović, *On graphs whose second largest eigenvalue does not exceed $\sqrt{2} - 1$* , *Univ. Beograd. Publ. Elektrotehn. Fak. Ser. Mat.* **4** (1993) 70–75.
- [13] S. Simić, *Complementary pairs of graphs with the second largest eigenvalue not exceeding $(\sqrt{5} - 1)/2$* , *Publ. Inst. Math. (Beograd) (N.S.)* **57(71)** (1995) 179–188.

- [14] S. Simić, *Some notes on graphs whose second largest eigenvalue is less than $(\sqrt{5} - 1)/2$* , Linear Multilinear Algebra **39** (1995) 59–71.
doi:10.1080/03081089508818380
- [15] S.K. Simić, M. Anđelić, C.M. da Fonseca and D. Živković, *Notes on the second largest eigenvalue of a graph*, Linear Algebra Appl. **465** (2015) 262–274.
doi:10.1016/j.laa.2014.09.032
- [16] S.K. Simić, D. Živković, M. Anđelić and C.M. da Fonseca, *Reflexive line graphs of trees*, J. Algebraic Combin. **43** (2016) 447–464.
doi:10.1007/s10801-015-0640-z
- [17] J.H. Smith, *Some properties of the spectrum of a graph*, in: Combin. Struct. Appl., (Gordon and Breach, New York, 1970) 403–406.
- [18] Z. Stanić, *Some graphs whose second largest eigenvalue does not exceed $\sqrt{2}$* , Linear Algebra Appl. **437** (2012) 1812–1820.
doi:10.1016/J.laa.2012.04.044

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