

## SIGNED COMPLETE GRAPHS WITH MAXIMUM INDEX

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*In the Memorial of Professor Slobodan Simić*

### Abstract

Let  $\Gamma = (G, \sigma)$  be a signed graph, where  $G$  is the underlying simple graph and  $\sigma : E(G) \rightarrow \{-, +\}$  is the sign function on the edges of  $G$ . The adjacency matrix of a signed graph has  $-1$  or  $+1$  for adjacent vertices, depending on the sign of the edges. It was conjectured that if  $\Gamma$  is a signed complete graph of order  $n$  with  $k$  negative edges,  $k < n - 1$  and  $\Gamma$  has maximum index, then negative edges form  $K_{1,k}$ . In this paper, we prove this conjecture if we confine ourselves to all signed complete graphs of order  $n$  whose negative edges form a tree of order  $k + 1$ . A  $[1, 2]$ -subgraph of  $G$  is a graph whose components are paths and cycles. Let  $\Gamma$  be a signed complete graph whose negative edges form a  $[1, 2]$ -subgraph. We show that the eigenvalues of  $\Gamma$  satisfy the following inequalities:

$$-5 \leq \lambda_n \leq \dots \leq \lambda_2 \leq 3.$$

**Keywords:** signed graph, complete graph, index.

**2010 Mathematics Subject Classification:** 05C22, 05C50.

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Received 4 June 2019  
Revised 26 November 2019  
Accepted 26 November 2019