

ON THE DISTANCE SPECTRAL RADIUS OF TREES WITH GIVEN DEGREE SEQUENCE

KENNETH DADEDZI

VALISOA RAZANAJATOVO MISANANTENAINA

AND

STEPHAN WAGNER

Department of Mathematical Sciences
Stellenbosch University
South Africa

e-mail: dadedzi@sun.ac.za
valisoa@sun.ac.za
swagner@sun.ac.za

This paper is dedicated to the memory of Slobodan Simić.

Abstract

We consider the problem of maximizing the distance spectral radius and a slight generalization thereof among all trees with some prescribed degree sequence. We prove in particular that the maximum of the distance spectral radius has to be attained by a caterpillar for any given degree sequence. The same holds true for the terminal distance matrix. Moreover, we consider a generalized version of the reverse distance matrix and also study its spectral radius for trees with given degree sequence. We prove that the spectral radius is always maximized by a greedy tree. This implies several corollaries, among them a “reversed” version of a conjecture of Stevanović and Ilić. Our results parallel similar theorems for the Wiener index and other invariants.

Keywords: distance matrix, spectral radius, tree, degree sequence.

2010 Mathematics Subject Classification: 05C05, 05C50, 05C12.

REFERENCES

- [1] A.T. Balaban, D. Mills, O. Ivanciuc and S.C. Basak, *Reverse Wiener indices*, Croat. Chem. Acta **73** (2000) 923–941.

- [2] R.B. Bapat, *Determinant of the distance matrix of a tree with matrix weights*, Linear Algebra Appl. **416** (2006) 2–7.
doi:10.1016/j.laa.2005.02.022
- [3] R.B. Bapat, *Graphs and Matrices* (Springer, 2010).
doi:10.1007/978-1-84882-981-7
- [4] R.B. Bapat, S.J. Kirkland and M. Neumann, *On distance matrices and Laplacians*, Linear Algebra Appl. **401** (2005) 193–209.
doi:10.1016/j.laa.2004.05.011
- [5] T. Bıykođlu and J. Leydold, *Graphs with given degree sequence and maximal spectral radius*, Electron. J. Combin. **15** (2008) #R119.
- [6] S.S. Bose, M. Nath and S. Paul, *Distance spectral radius of graphs with r pendent vertices*, Linear Algebra Appl. **435** (2011) 2828–2836.
doi:10.1016/j.laa.2011.04.041
- [7] S.S. Bose, M. Nath and S. Paul, *On the maximal distance spectral radius of graphs without a pendent vertex*, Linear Algebra Appl. **438** (2013) 4260–4278.
doi:10.1016/j.laa.2013.01.019
- [8] S.S. Bose, M. Nath and D. Sarma, *Maximal distance spectral radius of trees*, Discrete Math. Algorithms Appl. **11** (2019) 1950025.
doi:10.1142/S1793830919500253
- [9] E. ela, N.S. Schmuck, S. Wimer and G.J. Woeginger, *The Wiener maximum quadratic assignment problem*, Discrete Optim. **8** (2011) 411–416.
doi:10.1016/j.disopt.2011.02.002
- [10] K.L. Collins, *Distance Matrices of Trees* (Ph.D. Thesis, Massachusetts Institute of Technology, 1986).
- [11] M. Fischermann, A. Hoffmann, D. Rautenbach, L. Székely and L. Volkmann, *Wiener index versus maximum degree in trees*, Discrete Appl. Math. **122** (2002) 127–137.
doi:10.1016/S0166-218X(01)00357-2
- [12] M. Goubko, *On minimum terminal distance spectral radius of trees with given degree sequence*, (2015).
arXiv:1507.01733
- [13] R.L. Graham and H.O. Pollak, *On the addressing problem for loop switching*, Bell System Tech. J. **50** (1971) 2495–2519.
doi:10.1002/j.1538-7305.1971.tb02618.x
- [14] G.H. Hardy, J.E. Littlewood and G. Pólya, *Inequalities* (Cambridge University Press, 1952).
- [15] A. Heydari, *On extremal trees with respect to their terminal distance spectral radius*, Australas. J. Combin. **69** (2017) 159–168.
- [16] A. Ilić, *Distance spectral radius of trees with given matching number*, Discrete Appl. Math. **158** (2010) 1799–1806.
doi:10.1016/j.dam.2010.06.018

- [17] G. Indulal, *Sharp bounds on the distance spectral radius and the distance energy of graphs*, Linear Algebra Appl. **430** (2009) 106–113.
doi:10.1016/j.laa.2008.07.005
- [18] H. Lin and B. Zhou, *The distance spectral radius of graphs with given number of odd vertices*, Electron. J. Linear Algebra **31** (2016) 286–305.
doi:10.13001/1081-3810.2877
- [19] H. Lin and B. Zhou, *The distance spectral radius of trees*, Linear Multilinear Algebra **67** (2018) 370–390.
doi:10.1080/03081087.2017.1418830
- [20] W. Lin, Y. Zhang, Q. Chen, J. Chen, C. Ma and J. Chen, *Ordering trees by their distance spectral radii*, Discrete Appl. Math. **203** (2016) 106–110.
doi:10.1016/j.dam.2015.09.009
- [21] Z. Luo and B. Zhou, *On distance spectral radius of trees and fixed maximum degree*, Filomat **29** (2015) 2021–2026.
doi:10.2298/FIL1509021L
- [22] M. Nath and S. Paul, *On the distance spectral radius of trees*, Linear Multilinear Algebra **61** (2013) 847–855.
doi:10.1080/03081087.2012.711324
- [23] W. Ning, L. Ouyang and M. Lu, *Distance spectral radius of trees with fixed number of pendent vertices*, Linear Algebra Appl. **439** (2013) 2240–2249.
doi:10.1016/j.laa.2013.06.030
- [24] N.S. Schmuck, S.G. Wagner and H. Wang, *Greedy trees, caterpillars, and Wiener-type graph invariants*, MATCH Commun. Math. Comput. Chem. **68** (2012) 273–292.
- [25] D. Stevanović and A. Ilić, *Distance spectral radius of trees with fixed maximum degree*, Electron. J. Linear Algebra **20** (2010) 168–179.
doi:10.13001/1081-3810.1366
- [26] L.A. Székely, H. Wang and T. Wu, *The sum of the distances between the leaves of a tree and the semi-regular property*, Discrete Math. **311** (2011) 1197–1203.
doi:10.1016/j.disc.2010.06.005
- [27] H. Wang, *The extremal values of the Wiener index of a tree with given degree sequence*, Discrete Appl. Math. **156** (2008) 2647–2654.
doi:10.1016/j.dam.2007.11.005
- [28] Y. Wang, R. Xing, B. Zhou and F. Dong, *A note on distance spectral radius of trees*, Spec. Matrices **5** (2017) 296–300.
doi:10.1515/spma-2017-0021
- [29] Y. Wang and B. Zhou, *On distance spectral radius of graphs*, Linear Algebra Appl. **438** (2013) 3490–3503.
doi:10.1016/j.laa.2012.12.024
- [30] R. Xing, B. Zhou and F. Dong, *The effect of a graft transformation on distance spectral radius*, Linear Algebra Appl. **457** (2014) 261–275.
doi:10.1016/j.laa.2014.05.024

- [31] G. Yu, S. Guo and M. Zhai, *Distance spectral radius of a tree with given diameter*, *Ars Combin.* **134** (2017) 351–362.
- [32] X.-D. Zhang, *The Laplacian spectral radii of trees with given degree sequences*, *Discrete Math.* **308** (2008) 3143–3150.
doi:10.1016/j.disc.2007.06.017
- [33] X.-D. Zhang, Q.-Y. Xiang, L.-Q. Xu and R.-Y. Pan, *The Wiener index of trees with given degree sequences*, *MATCH Commun. Math. Comput. Chem.* **60** (2008) 623–644.

Received 30 April 2019

Revised 9 July 2019

Accepted 18 July 2019