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THE NUMBER OF P-VERTICES OF SINGULAR ACYCLIC MATRICES: AN INVERSE PROBLEM

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Dedicated to the memory of Prof. Slobodan K. Simić. In honour of Prof. Slobodan K. Simić for his decisive contributions to spectral graph theory.

Abstract

Let A be a real symmetric matrix. If after we delete a row and a column of the same index, the nullity increases by one, we call that index a P-vertex of A. When A is an $n \times n$ singular acyclic matrix, it is known that the maximum number of P-vertices is $n-2$. If T is the underlying tree of A, we will show that for any integer number $k \in \{0, 1, \ldots, n-2\}$, there is a (singular) matrix whose graph is T and with k P-vertices. We will provide illustrative examples.

Keywords: trees, acyclic matrices, singular, multiplicity of eigenvalues, P-set, P-vertices.

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