

EXISTENCE OF REGULAR NUT GRAPHS FOR DEGREE AT MOST 11

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Dedicated to the memory of Slobodan Simić.

Abstract

A nut graph is a singular graph with one-dimensional kernel and corresponding eigenvector with no zero elements. The problem of determining the orders n for which d -regular nut graphs exist was recently posed by Gauci, Pisanski and Sciriha. These orders are known for $d \leq 4$. Here we solve the problem for all remaining cases $d \leq 11$ and determine the complete lists of all d -regular nut graphs of order n for small values of d and n . The existence or non-existence of small regular nut graphs is determined by a computer search. The main tool is a construction that produces, for any d -regular nut graph of order n , another d -regular nut graph of order $n + 2d$. If we are given a sufficient number of d -regular nut graphs of consecutive orders, called seed graphs, this construction may be applied in such a way that the existence of all d -regular nut graphs of higher orders is established. For even d the orders n are indeed consecutive, while for odd d the orders n are consecutive even numbers. Furthermore, necessary conditions for combinations of order and degree for vertex-transitive nut graphs are derived.

Keywords: nut graph, core graph, regular graph, nullity.

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REFERENCES

- [1] G. Brinkmann, K. Coolsaet, J. Goedgebeur and H. Mélot, *House of graphs: A database of interesting graphs*, Discrete Appl. Math. **161** (2013) 311–314.
doi:10.1016/j.dam.2012.07.018
- [2] G. Brinkmann, J. Goedgebeur and B.D. McKay, *Generation of cubic graphs*, Discrete Math. Theor. Comput. Sci. **13** (2011) 69–80.
- [3] K. Coolsaet, P.W. Fowler and J. Goedgebeur, homepage of Nutgen.
<http://caagt.ugent.be/nutgen/>
- [4] K. Coolsaet, P.W. Fowler and J. Goedgebeur, *Generation and properties of nut graphs*, MATCH Commun. Math. Comput. Chem. **80** (2018) 423–444.
- [5] P.W. Fowler, B.T. Pickup, T.Z. Todorova, M. Borg and I. Sciriha, *Omni-conducting and omni-insulating molecules*, J. Chem. Phys. **140** (2014) 054115.
doi:10.1063/1.4863559
- [6] J.B. Gauci, T. Pisanski and I. Sciriha, *Existence of regular nut graphs and the Fowler construction*, (2019).
arXiv preprint arXiv:1904.02229
- [7] D. Holt and G.F. Royle, *A census of small transitive groups and vertex-transitive graphs*, J. Symbolic Comput. (2019), in press.
doi:10.1016/j.jsc.2019.06.006

- [8] B.D. McKay and G.F. Royle, *The transitive graphs with at most 26 vertices*, Ars Combin. **30** (1990) 161–176.
- [9] M. Meringer, *Fast generation of regular graphs and construction of cages*, J. Graph Theory **30** (1999) 137–146.
doi:10.1002/(SICI)1097-0118(199902)30:2<137::AID-JGT7>3.0.CO;2-G
- [10] I. Sciriha, *On the construction of graphs of nullity one*, Discrete Math. **181** (1998) 193–211.
doi:10.1016/S0012-365X(97)00036-8
- [11] I. Sciriha, *On the rank of graphs*, in: Combinatorics, Graph Theory and Algorithms, Vol. II, Y. Alavi, D.R. Lick and A. Schwenk (Ed(s)), (Springer, Michigan, 1999) 769–778.
- [12] I. Sciriha, *A characterization of singular graphs*, Electron. J. Linear Algebra **16** (2007) 451–462.
doi:10.13001/1081-3810.1215
- [13] I. Sciriha, *Coalesced and embedded nut graphs in singular graphs*, Ars Math. Contemp. **1** (2008) 20–31.
doi:10.26493/1855-3974.20.7cc
- [14] I. Sciriha and I. Gutman, *Nut graphs: maximally extending cores*, Util. Math. **54** (1998) 257–272.

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