Discussiones Mathematicae
Graph Theory 40 (2020) 559–575
doi:10.7151/dmgt.2268

ON THE α-SPECTRAL RADIUS OF UNIFORM HYPERGRAPHS

Haiyan Guo and Bo Zhou[1]

School of Mathematical Sciences
South China Normal University
Guangzhou 510631, P.R. China

e-mail: ghaiyan0705@163.com
zhoubo@scnu.edu.cn

Abstract

For $0 \leq \alpha < 1$ and a uniform hypergraph G, the α-spectral radius of G is the largest H-eigenvalue of $\alpha\mathcal{D}(G)+(1-\alpha)\mathcal{A}(G)$, where $\mathcal{D}(G)$ and $\mathcal{A}(G)$ are the diagonal tensor of degrees and the adjacency tensor of G, respectively. We give upper bounds for the α-spectral radius of a uniform hypergraph, propose some transformations that increase the α-spectral radius, and determine the unique hypergraphs with maximum α-spectral radius in some classes of uniform hypergraphs.

Keywords: α-spectral radius, α-Perron vector, adjacency tensor, uniform hypergraph, extremal hypergraph.

2010 Mathematics Subject Classification: 05C50, 05C65.

References

[1] J. Cooper and A. Dutle, *Spectra of uniform hypergraphs*, Linear Algebra Appl. **436** (2012) 3268–3292.
doi:10.1016/j.laa.2011.11.018

[2] Y. Fan, Y. Tan, X. Peng and A. Liu, *Maximizing spectral radii of uniform hypergraphs with few edges*, Discuss. Math. Graph Theory **36** (2016) 845–856.
doi:10.7151/dmgt.1906

[3] S. Friedland, S. Gaubert and L. Han, *Perron-Frobenius theorem for nonnegative multilinear forms and extension*, Linear Algebra Appl. **438** (2013) 738–749.
doi:10.1016/j.laa.2011.02.042

[1]Corresponding author.

[4] H. Guo and B. Zhou, *On the spectral radius of uniform hypertrees*, Linear Algebra Appl. **558** (2018) 236–249.
doi:10.1016/j.laa.2018.07.035

[5] H. Guo and B. Zhou, *On the α-spectral radius of graphs.*
ArXiv:1805.03456

[6] S. Hu, L. Qi and J. Xie, *The largest Laplacian and signless Laplacian H-eigenvalues of a uniform hypergraph*, Linear Algebra Appl. **469** (2015) 1–27.
doi:10.1016/j.laa.2014.11.020

[7] C. Li, Z. Chen and Y. Li, *A new eigenvalue inclusion set for tensors and its applications*, Linear Algebra Appl. **481** (2015) 36–53.
doi:10.1016/j.laa.2015.04.023

[8] H. Li, J. Shao and L. Qi, *The extremal spectral radii of k-uniform supertrees*, J Comb. Optim. **32** (2016) 741–764.
doi:10.1007/s10878-015-9896-4

[9] H. Li, J. Zhou and C. Bu, *Principal eigenvectors and spectral radii of uniform hypergraphs*, Linear Algebra Appl. **544** (2018) 273–285.
doi:10.1016/j.laa.2018.01.017

[10] L.-H. Lim, *Singular values and eigenvalues of tensors: a variational approach*, in: Proceedings of the First IEEE International Workshop on Computational Advances of Multi-Sensor Adaptive Processing, Puerto Vallarta (2005) 129–132.
doi:10.110SICAMAP.2005.1574201

[11] H. Lin, H. Guo and B. Zhou, *On the α-spectral radius of irregular uniform hypergraphs*, Linear Multilinear Algebra, 2019.
doi:10.1080/03081087.2018.1502253

[12] H. Lin, B. Mo, B. Zhou and W. Weng, *Sharp bounds for ordinary and signless Laplacian spectral radii of uniform hypergraphs*, Appl. Math. Comput. **285** (2016) 217–227.
doi:10.1016/j.amc.2016.03.016

[13] H. Lin, B. Zhou and B. Mo, *Upper bounds for H- and Z-spectral radii of uniform hypergraphs*, Linear Algebra Appl. **510** (2016) 205–221.
doi:10.1016/j.laa.2016.08.009

[14] V. Nikiforov, *Merging the A- and Q-spectral theories*, Appl. Anal. Discrete Math. **11** (2017) 81–107.
doi:10.2298/AADM1701081N

[15] V. Nikiforov, G. Pastén, O. Rojo and R.L. Soto, *On the A_α-spectra of trees*, Linear Algebra Appl. **520** (2017) 286–305.
doi:10.1016/j.laa.2017.01.029

[16] C. Ouyang, L. Qi and X. Yuan, *The first few unicyclic and bicyclic hypergraphs with largest spectral radii*, Linear Algebra Appl. **527** (2017) 141–162.
doi:10.1016/j.laa.2017.04.008

[17] K.J. Pearson and T. Zhang, *On spectral hypergraph theory of the adjacency tensor*, Graphs Combin. **30** (2014) 1233–1248.
doi:10.1007/s00373-013-1340-x

[18] L. Qi, *Eigenvalues of a real supersymmetric tensor*, J. Symbolic Comput. **40** (2005) 1302–1324.
doi:10.1016/j.jsc.2005.05.007

[19] L. Qi, *Symmetric nonnegative tensors and copositive tensors*, Linear Algebra Appl. **439** (2013) 228–238.
doi:10.1016/j.laa.2013.03.015

[20] L. Qi, H^+*-eigenvalues of Laplacian and signless Laplacian tensors*, Commun. Math. Sci. **12** (2014) 1045–1064.
doi:10.4310/CMS.2014.v12.n6.a3

[21] L. Qi, J. Shao and Q. Wang, *Regular uniform hypergraphs, s-cycles, s-paths and their largest Laplacian H-eigenvalues*, Linear Algebra Appl. **443** (2014) 215–227.
doi:10.1016/j.laa.2013.11.008

[22] J.Y. Shao, *A general product of tensors with applications*, Linear Algebra Appl. **439** (2013) 2350–2366.
doi:10.1016/j.laa.2013.07.010

[23] S.K. Simić and B. Zhou, *Indices of trees with a prescribed diameter*, Appl. Anal. Discrete Math. **1** (2007) 446–454.
doi:10.2298/AADM0702446S

[24] P. Xiao, L. Wang and Y. Lu, *The maximum spectral radii of uniform supertrees with given degree sequences*, Linear Algebra Appl. **523** (2017) 33–45.
doi:10.1016/j.laa.2017.02.018

[25] P. Xiao, L. Wang and Y. Du, *The first two largest spectral radii of uniform supertrees with given diameter*, Linear Algebra Appl. **536** (2018) 103–119.
doi:10.1016/j.laa.2017.09.009

[26] P. Xiao and L. Wang, *The maximum spectral radius of uniform hypergraphs with given number of pendant edges*, Linear Multilinear Algebra.
doi:10.1080/03081087.2018.1453471

[27] Y. Yang and Q. Yang, *Further results for Perron-Frobenius theorem for nonegative tensors*, SIAM J. Matrix Anal. Appl. **31** (2010) 2517–2530.
doi:10.1137/090778766

[28] X. Yuan, J. Shao and H. Shan, *Ordering of some uniform supertrees with larger spectral radii*, Linear Algebra Appl. **495** (2016) 206–222.
doi:10.1016/j.laa.2016.01.031

[29] X. Yuan, M. Zhang and M. Lu, *Some upper bounds on the eigenvalues of uniform hypergraphs*, Linear Algebra Appl. **484** (2015) 540–549.
doi:10.1016/j.laa.2015.06.023

[30] J. Zhang and J. Li, *The maximum spectral radius of k-uniform hypergraphs with r pendent vertices*, Linear Multilinear Algebra **67** (2019) 1062–1073. doi:10.1080/03081087.2018.1442811

[31] J. Zhou, L. Sun, W. Wang and C. Bu, *Some spectral properties of uniform hypergraphs*, Electron. J. Combin. **21** (2014) #P4.24.

Received 14 August 2018
Revised 24 April 2019
Accepted 24 April 2019