

TURÁN'S THEOREM IMPLIES STANLEY'S BOUND

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Dedicated to the memory of Slobodan K. Simić

Abstract

Let G be a graph with m edges and let ρ be the largest eigenvalue of its adjacency matrix. It is shown that

$$\rho \leq \sqrt{2 \left(1 - \left[1/2 + \sqrt{2m + 1/4} \right]^{-1} \right) m},$$

improving the well-known bound of Stanley. Moreover, writing ω for the clique number of G and W_k for the number of its walks on k vertices, it is shown that the sequence

$$\left\{ \left((1 - 1/\omega) W_{2^k} \right)^{1/2^k} \right\}_{k=1}^{\infty}$$

is nonincreasing and converges to ρ .

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