

NOTE

A SHORT PROOF FOR A LOWER BOUND ON THE ZERO FORCING NUMBER

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AND

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Abstract

We provide a short proof of a conjecture of Davila and Kenter concerning a lower bound on the zero forcing number $Z(G)$ of a graph G . More specifically, we show that $Z(G) \geq (g - 2)(\delta - 2) + 2$ for every graph G of girth g at least 3 and minimum degree δ at least 2.

Keywords: zero forcing, girth, Moore bound.

2010 Mathematics Subject Classification: 05C69.

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Received 31 May 2017
Revised 26 February 2018
Accepted 26 February 2018