INCIDENCE COLORING—COLD CASES

FRANTIŠEK KARDOŠ¹, MÁRIA MACEKOVÁ²

MARTINA MOCKOVČIAKOVÁ³, ÉRIC SOPENA¹

AND

ROMAN SOTÁK²

¹Univ. Bordeaux, Bordeaux INP, CNRS, LaBRI, UMR5800, F-33400 Talence, France

²Institute of Mathematics
P.J. Šafárik University in Košice
Jesenná 5, 04001 Košice, Slovakia

³European Centre of Excellence NTIS
University of West Bohemia
Pilsen, Czech Republic

e-mail: frantisek.kardos@labri.fr
maria.macekova@upjs.sk
mmockov@ntis.zcu.cz
eric.sopena@labri.fr
roman.sotak@upjs.sk

Abstract

An incidence in a graph G is a pair \((v, e)\) where \(v\) is a vertex of G and \(e\) is an edge of G incident to \(v\). Two incidences \((v, e)\) and \((u, f)\) are adjacent if at least one of the following holds: (i) \(v = u\), (ii) \(e = f\), or (iii) edge \(vu\) is from the set \(\{e, f\}\). An incidence coloring of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The minimum number of colors needed for incidence coloring of a graph is called the incidence chromatic number.

It was proved that at most \(\Delta(G) + 5\) colors are enough for an incidence coloring of any planar graph G except for \(\Delta(G) = 6\), in which case at most 12 colors are needed. It is also known that every planar graph G with girth at least 6 and \(\Delta(G) \geq 5\) has incidence chromatic number at most \(\Delta(G) + 2\).

In this paper we present some results on graphs regarding their maximum degree and maximum average degree. We improve the bound for planar graphs with \(\Delta(G) = 6\). We show that the incidence chromatic number is at
most $\Delta(G) + 2$ for any graph $G$ with $\text{mad}(G) < 3$ and $\Delta(G) = 4$, and for any graph with $\text{mad}(G) < \frac{10}{3}$ and $\Delta(G) \geq 8$.

**Keywords**: incidence coloring, incidence chromatic number, planar graph, maximum average degree.

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REFERENCES


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