INCIDENCE COLORING—COLD CASES

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Abstract

An incidence in a graph G is a pair \((v, e)\) where v is a vertex of G and e is an edge of G incident to v. Two incidences \((v, e)\) and \((u, f)\) are adjacent if at least one of the following holds: (i) \(v = u\), (ii) \(e = f\), or (iii) edge \(vu\) is from the set \{e, f\}. An incidence coloring of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The minimum number of colors needed for incidence coloring of a graph is called the incidence chromatic number.

It was proved that at most \(\Delta(G) + 5\) colors are enough for an incidence coloring of any planar graph G except for \(\Delta(G) = 6\), in which case at most 12 colors are needed. It is also known that every planar graph G with girth at least 6 and \(\Delta(G) \geq 5\) has incidence chromatic number at most \(\Delta(G) + 2\).

In this paper we present some results on graphs regarding their maximum degree and maximum average degree. We improve the bound for planar graphs with \(\Delta(G) = 6\). We show that the incidence chromatic number is at
most $\Delta(G) + 2$ for any graph $G$ with $\text{mad}(G) < 3$ and $\Delta(G) = 4$, and for any graph with $\text{mad}(G) < \frac{14}{3}$ and $\Delta(G) \geq 8$.

**Keywords:** incidence coloring, incidence chromatic number, planar graph, maximum average degree.

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References


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