Discussiones Mathematicae
Graph Theory 40 (2020) 345–354
doi:10.7151/dmgt.2140

INCIDENCE COLORING—COLD CASES

František Kardoš[1], Mária Maceková[2]

Martina Mockovčiaková[3], Éric Sopena[1]

and

Roman Soták[2]

[1] *Univ. Bordeaux, Bordeaux INP, CNRS, LaBRI,*
UMR5800, F-33400 Talence, France

[2] *Institute of Mathematics*
P.J. Šafárik University in Košice
Jesenná 5, 04001 Košice, Slovakia

[3] *European Centre of Excellence NTIS*
University of West Bohemia
Pilsen, Czech Republic

e-mail: frantisek.kardos@labri.fr
maria.macekova@upjs.sk
mmockov@ntis.zcu.cz
eric.sopena@labri.fr
roman.sotak@upjs.sk

Abstract

An *incidence* in a graph G is a pair (v, e) where v is a vertex of G and e is an edge of G incident to v. Two incidences (v, e) and (u, f) are adjacent if at least one of the following holds: (i) $v = u$, (ii) $e = f$, or (iii) edge vu is from the set $\{e, f\}$. An *incidence coloring* of G is a coloring of its incidences assigning distinct colors to adjacent incidences. The minimum number of colors needed for incidence coloring of a graph is called the *incidence chromatic number*.

It was proved that at most $\Delta(G) + 5$ colors are enough for an incidence coloring of any planar graph G except for $\Delta(G) = 6$, in which case at most 12 colors are needed. It is also known that every planar graph G with girth at least 6 and $\Delta(G) \geq 5$ has incidence chromatic number at most $\Delta(G) + 2$.

In this paper we present some results on graphs regarding their maximum degree and maximum average degree. We improve the bound for planar graphs with $\Delta(G) = 6$. We show that the incidence chromatic number is at most $\Delta(G)+2$ for any graph G with $\mathrm{mad}(G)<3$ and $\Delta(G)=4$, and for any graph with $\mathrm{mad}(G)<\frac{10}{3}$ and $\Delta(G)\geq 8$.

Keywords: incidence coloring, incidence chromatic number, planar graph, maximum average degree.

2010 Mathematics Subject Classification: 05C15.

References

[1] M. Bonamy, B. Lévêque and A. Pinlou, 2-*distance coloring of sparse graphs*, J. Graph Theory **77** (2014) 190–218.
doi:10.1002/jgt.21782

[2] R.A. Brualdi and J.J. Quinn Massey, *Incidence and strong edge colorings of graphs*, Discrete Math. **122** (1993) 51–58.
doi:10.1016/0012-365X(93)90286-3

[3] B. Guiduli, *On incidence coloring and star arboricity of graphs*, Discrete Math. **163** (1997) 275–278.
doi:10.1016/0012-365X(95)00342-T

[4] S.L. Hakimi, J. Mitchem and E. Schmeichel, *Star arboricity of graphs*, Discrete Math. **149** (1996) 93–98.
doi:10.1016/0012-365X(94)00313-8

[5] M. Hosseini Dolama, É. Sopena and X. Zhu, *Incidence coloring of k-degenerated graphs*, Discrete Math. **283** (2004) 121–128.
doi:10.1016/j.disc.2004.01.015

[6] M. Hosseini Dolama and É. Sopena, *On the maximum average degree and the incidence chromatic number of a graph*, Discrete Math. Theor. Comput. Sci. **7** (2005) 203–216.

[7] D.P. Sanders and Y. Zhao, *Planar graphs of maximum degree seven are class* I, J. Combin. Theory Ser. B **83** (2001) 201–212.
doi:10.1006/jctb.2001.2047

[8] D. Yang, *Fractional incidence coloring and star arboricity of graphs*, Ars Combin. **105** (2012) 213–224.

Received 9 October 2017

Revised 25 April 2018

Accepted 26 April 2018