

## STRONG GEODETIC PROBLEM IN NETWORKS

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### Abstract

In order to model certain social network problems, the strong geodesic problem and its related invariant, the strong geodesic number, are introduced. The problem is conceptually similar to the classical geodesic problem but seems intrinsically more difficult. The strong geodesic number is compared with the geodesic number and with the isometric path number. It is determined for several families of graphs including Apollonian networks. Applying Sierpiński graphs, an algorithm is developed that returns a minimum path cover of Apollonian networks corresponding to the strong geodesic number. It is also proved that the strong geodesic problem is NP-complete.

**Keywords:** geodesic problem, strong geodesic problem, Apollonian networks, Sierpiński graphs, computational complexity.

**2010 Mathematics Subject Classification:** 05C12, 05C70, 68Q17.

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Received 17 July 2017  
 Revised 19 March 2018  
 Accepted 19 March 2018