Discussiones Mathematicae
Graph Theory 40 (2020) 297–305
doi:10.7151/dmgt.2141

LOWER BOUND ON THE NUMBER OF HAMILTONIAN CYCLES OF GENERALIZED PETERSEN GRAPHS

Weihua Lu

College of Arts and Sciences
Shanghai Maritime University,
Shanghai, 201306, P.R. China

e-mail: lwh8797@163.com

Chao Yang[1]

School of Mathematics, Physics and Statistics
Shanghai University of Engineering Science,
Shanghai, 201620, P.R. China

e-mail: yangchaomath0524@163.com

and

Han Ren

Department of Mathematics
East China Normal University,
Shanghai, 200241, P.R. China

Shanghai Key Laboratory of PMMP
Shanghai, 200241, P.R. China

e-mail: hren@math.ecnu.edu.cn

Abstract

In this paper, we investigate the number of Hamiltonian cycles of a generalized Petersen graph $P(N,k)$ and prove that

$$\Psi\big(P(N,3)\big) \geqslant N \cdot \alpha_N,$$

where $\Psi\big(P(N,3)\big)$ is the number of Hamiltonian cycles of $P(N,3)$ and α_N satisfies that for any $\varepsilon > 0$, there exists a positive integer M such that when $N > M$,

$$\left((1-\varepsilon)\frac{(1-r^3)}{6r^3+5r^2+3}\right)\left(\frac{1}{r}\right)^{N+2} < \alpha_N < \left((1+\varepsilon)\frac{(1-r^3)}{6r^3+5r^2+3}\right)\left(\frac{1}{r}\right)^{N+2},$$

[1]The corresponding author.

where $\frac{1}{r} = \max\left\{\left|\frac{1}{r_j}\right| : j = 1, 2, \ldots, 6\right\}$ and each r_j is a root of equation $x^6 + x^5 + x^3 - 1 = 0$, $r \approx 0.782$. This shows that $\Psi\big(P(N,3)\big)$ is exponential in N and also deduces that the number of 1-factors of $P(N,3)$ is exponential in N.

Keywords: generalized Petersen graph, Hamiltonian cycle, partition number, 1-factor.

2010 Mathematics Subject Classification: 05C30, 05C45, 05C70.

References

[1] B. Alspach, *The classification of Hamiltonian generalized Petersen graphs*, J. Combin. Theory Ser. B **34** (1983) 293–312.
doi:10.1016/0095-8956(83)90042-4

[2] K. Bannai, *Hamiltonian cycles in generalized Petersen graphs*, J. Combin. Theory Ser. B **24** (1978) 181–188.
doi:10.1016/0095-8956(78)90019-9

[3] J.A. Bondy, *Variations on the Hamiltonian theme*, Canad. Math. Bull. **15** (1972) 57–62.
doi:10.4153/CMB-1972-012-3

[4] J.A. Bondy and U.S.R. Murty, Graph Theory (Springer, 2008).

[5] F. Castagna and G. Prins, *Every generalized Petersen graph has a Tait coloring*, Pacific J. Math. **40** (1972) 53–58.
doi:10.2140/pjm.1972.40.53

[6] C. Cooper and A.M. Frieze, *On the number of Hamiltonian cycles in a random graph*, J. Graph Theory **13** (1989) 719–735.
doi:10.1002/jgt.3190130608

[7] M.R. Garey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness (Freeman, San Francisco, 1979).

[8] C.H. Papadimitriou, Computational Complexity (Addison-Wesley, Reading, MA, 1994).

[9] G.N. Robertson, Graphs under Girth, Valency, and Connectivity Constraints, PhD. Thesis (University of Waterloo, Ontario, Canada, 1968).

[10] A. Schwenk, *Enumeration of Hamiltonian cycles in certain generalized Petersen graphs*, J. Combin. Theory Ser. B **47** (1989) 53–59.
doi:10.1016/0095-8956(89)90064-6

[11] A. Thomason, *Cubic graphs with three Hamiltonian cycles are not always uniquely edge colorable*, J. Graph Theory **6** (1982) 219–221.
doi:10.1002/jgt.3190060218

[12] A.G. Thomason, *Hamiltonian cycles and uniquely edge colourable graphs*, Ann. Discrete Math. **3** (1978) 259–268. doi:10.1016/S0167-5060(08)70511-9

[13] W.T. Tutte, *On Hamiltonian circuits*, J. Lond. Math. Soc. (2) **21** (1946) 98–101. doi:10.1112/jlms/s1-21.2.98

[14] M.E. Watkins, *A theorem on tait colorings with an application to the generalized Petersen graphs*, J. Combin. Theory **6** (1969) 152–164. doi:10.1016/S0021-9800(69)80116-X

Received 11 April 2017
Revised 17 March 2018
Accepted 19 March 2018