LOWER BOUND ON THE NUMBER OF HAMILTONIAN CYCLES OF GENERALIZED PETERSEN GRAPHS

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Abstract

In this paper, we investigate the number of Hamiltonian cycles of a generalized Petersen graph \( P(N,k) \) and prove that

\[ \Psi(P(N,3)) \geq N \cdot \alpha_N, \]

where \( \Psi(P(N,3)) \) is the number of Hamiltonian cycles of \( P(N,3) \) and \( \alpha_N \) satisfies that for any \( \epsilon > 0 \), there exists a positive integer \( M \) such that when \( N > M \),

\[ \left( 1 - \epsilon \right) \frac{(1 - r^3)}{6r^3 + 5r^2 + 3} \left( \frac{1}{r} \right)^{N+2} < \alpha_N < \left( 1 + \epsilon \right) \frac{(1 - r^3)}{6r^3 + 5r^2 + 3} \left( \frac{1}{r} \right)^{N+2}, \]

\( ^1 \text{The corresponding author.} \)
where \( \frac{1}{r} = \max \left\{ \left| \frac{1}{r_j} \right| : j = 1, 2, \ldots, 6 \right\} \) and each \( r_j \) is a root of equation \( x^6 + x^5 + x^3 - 1 = 0 \), \( r \approx 0.782 \). This shows that \( \Psi(P(N, 3)) \) is exponential in \( N \) and also deduces that the number of 1-factors of \( P(N, 3) \) is exponential in \( N \).

**Keywords:** generalized Petersen graph, Hamiltonian cycle, partition number, 1-factor.

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### References


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