LOWER BOUND ON THE NUMBER OF HAMILTONIAN CYCLES OF GENERALIZED PETERSEN GRAPHS

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Abstract

In this paper, we investigate the number of Hamiltonian cycles of a generalized Petersen graph \( P(N, k) \) and prove that

\[
\Psi(P(N, 3)) \geq N \cdot \alpha_N,
\]

where \( \Psi(P(N, 3)) \) is the number of Hamiltonian cycles of \( P(N, 3) \) and \( \alpha_N \) satisfies that for any \( \varepsilon > 0 \), there exists a positive integer \( M \) such that when \( N > M \),

\[
\left(1 - \varepsilon\right) \left(\frac{1 - r^3}{6r^3 + 5r^2 + 3}\right) \left(\frac{1}{r}\right)^{N+2} < \alpha_N < \left(1 + \varepsilon\right) \left(\frac{1 - r^3}{6r^3 + 5r^2 + 3}\right) \left(\frac{1}{r}\right)^{N+2},
\]

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where $\frac{1}{r} = \max \left\{ \frac{1}{r_j} : j = 1, 2, \ldots, 6 \right\}$ and each $r_j$ is a root of equation $x^6 + x^5 + x^3 - 1 = 0$, $r \approx 0.782$. This shows that $\Psi(P(N, 3))$ is exponential in $N$ and also deduces that the number of 1-factors of $P(N, 3)$ is exponential in $N$.

**Keywords:** generalized Petersen graph, Hamiltonian cycle, partition number, 1-factor.

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**References**


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