

## NICHE HYPERGRAPHS OF PRODUCTS OF DIGRAPHS

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### Abstract

If  $D = (V, A)$  is a digraph, its *niche hypergraph*  $N\mathcal{H}(D) = (V, \mathcal{E})$  has the edge set  $\mathcal{E} = \{e \subseteq V \mid |e| \geq 2 \wedge \exists v \in V : e = N_D^-(v) \vee e = N_D^+(v)\}$ . Niche hypergraphs generalize the well-known niche graphs and are closely related to competition hypergraphs as well as common enemy hypergraphs. For several products  $D_1 \circ D_2$  of digraphs  $D_1$  and  $D_2$ , we investigate the relations between the niche hypergraphs of the factors  $D_1$ ,  $D_2$  and the niche hypergraph of their product  $D_1 \circ D_2$ .

**Keywords:** niche hypergraph, product of digraphs, competition hypergraph.

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