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ROMAN {2}-BONDAGE NUMBER OF A GRAPH

Ahmad Moradi, Doost Ali Mojdeh

and

Omid Sharifi

Department of Mathematical Sciences
University of Mazandaran, Babolsar, Iran

e-mail: a.moradi@umz.ac.ir
damojdeh@umz.ac.ir
omid.sharifi1365@gmail.com

Abstract

For a given graph $G = (V, E)$, a Roman $\{2\}$-dominating function $f : V(G) \to \{0, 1, 2\}$ has the property that for every vertex u with $f(u) = 0$, either u is adjacent to a vertex assigned 2 under f, or is adjacent to at least two vertices assigned 1 under f. The Roman $\{2\}$-domination number of G, $\gamma_{\{R2\}}(G)$, is the minimum of $\sum_{u \in V(G)} f(u)$ over all such functions. In this paper, we initiate the study of the problem of finding Roman $\{2\}$-bondage number of G. The Roman $\{2\}$-bondage number of G, $b_{\{R2\}}$, is defined as the cardinality of a smallest edge set $E' \subseteq E$ for which $\gamma_{\{R2\}}(G - E') > \gamma_{\{R2\}}(G)$. We first demonstrate complexity status of the problem by proving that the problem is NP-Hard. Then, we derive useful parametric as well as fixed upper bounds on the Roman $\{2\}$-bondage number of G. Specifically, it is known that the Roman bondage number of every planar graph does not exceed 15 (see [S. Akbari, M. Khatirinejad and S. Qajar, *A note on the Roman bondage number of planar graphs*, Graphs Combin. 29 (2013) 327–331]). We show that same bound will be preserved while computing the Roman $\{2\}$-bondage number of such graphs. The paper is then concluded by computing exact value of the parameter for some classes of graphs.

Keywords: domination, Roman $\{2\}$-domination, Roman $\{2\}$-bondage number.

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References

[1] S. Akbari, M. Khatirinejad and S. Qajar, *A note on the Roman bondage number of planar graphs*, Graphs Combin. **29** (2013) 327–331. doi:10.1007/s00373-011-1129-8

[2] A. Bahremandpour, F.-T. Hu, S.M. Sheikholeslami and J.-M. Xu, *On the Roman bondage number of a graph*, Discrete Math. Algorithms Appl. **5** (2013) #1350001. doi:10.1142/S1793830913500018

[3] E.W. Chambers, B. Kinnersley, N. Prince and D.B. West, *Extremal problems for Roman domination*, SIAM J. Discrete Math. **23** (2009) 1575–1586. doi:10.1137/070699688

[4] M. Chellali, T.W. Haynes, S.T. Hedetniemi and A.A. McRae, *Roman* {2}-*domination*, Discrete Appl. Math. **204** (2016) 22–28. doi:10.1016/j.dam.2015.11.013

[5] E.J. Cockayne, P.M. Dreyer Jr., S.M. Hedetniemi and S.T. Hedetniemi, *Roman domination in graphs*, Discrete Math. **278** (2004) 11–22. doi:10.1016/j.disc.2003.06.004

[6] J.F. Fink, M.S. Jacobson, L.F. Kinch and J. Roberts, *The bondage number of a graph*, Discrete Math. **86** (1990) 47–57. doi:10.1016/0012-365X(90)90348-L

[7] M.R. Garey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness (W.H. Freeman, 1979).

[8] J. Kleinberg and E. Tardos, Algorithm Design (Pearson Education, India, 2006).

[9] P. Roushini Leely Pushpam and T.N.M. Malini Mai, *Weak roman domination in graphs*, Discuss. Math. Graph Theory **31** (2011) 115–128. doi:10.7151/dmgt.1532

[10] N. Jafari Rad and L. Volkmann, *Roman bondage in graphs*, Discuss. Math. Graph Theory **31** (2011) 763–773. doi:10.7151/dmgt.1578

[11] M. Krzywkowski, 2-*bondage in graphs*, Int. J. Comput. Math. **90** (2013) 1358–1365. doi:10.1080/00207160.2012.752817

[12] T. Turaci, *On the average lower bondage number of a graph*, RAIRO Oper. Res. **50** (2016) 1003–1012. doi:10.1051/ro/2015062

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