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NEIGHBOR SUM DISTINGUISHING TOTAL CHROMATIC NUMBER OF PLANAR GRAPHS WITHOUT 5-CYCLES[1]

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Abstract

For a given graph $G = (V(G), E(G))$, a proper total coloring $\phi : V(G) \cup E(G) \to \{1, 2, \ldots, k\}$ is neighbor sum distinguishing if $f(u) \neq f(v)$ for each edge $uv \in E(G)$, where $f(v) = \sum_{uv \in E(G)} \phi(uv) + \phi(v)$, $v \in V(G)$. The smallest integer k in such a coloring of G is the neighbor sum distinguishing total chromatic number, denoted by $\chi''_\Sigma(G)$. Pilśniak and Woźniak first introduced this coloring and conjectured that $\chi''_\Sigma(G) \leq \Delta(G) + 3$ for any graph with maximum degree $\Delta(G)$. In this paper, by using the discharging method, we prove that for any planar graph G without 5-cycles, $\chi''_\Sigma(G) \leq \max\{\Delta(G) + 2, 10\}$. The bound $\Delta(G) + 2$ is sharp. Furthermore, we get the exact value of $\chi''_\Sigma(G)$ if $\Delta(G) \geq 9$.

Keywords: neighbor sum distinguishing total coloring, discharging method, planar graph.

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