

ON TOTAL H -IRREGULARITY STRENGTH OF THE DISJOINT UNION OF GRAPHS

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Abstract

A simple graph G admits an H -covering if every edge in $E(G)$ belongs to at least to one subgraph of G isomorphic to a given graph H . For the subgraph $H \subseteq G$ under a total k -labeling we define the associated H -weight as the sum of labels of all vertices and edges belonging to H . The total k -labeling is called the H -irregular total k -labeling of a graph G admitting

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an H -covering if all subgraphs of G isomorphic to H have distinct weights. The *total H -irregularity strength* of a graph G is the smallest integer k such that G has an H -irregular total k -labeling.

In this paper, we estimate lower and upper bounds on the total H -irregularity strength for the disjoint union of multiple copies of a graph and the disjoint union of two non-isomorphic graphs. We also prove the sharpness of the upper bounds.

Keywords: H -covering, H -irregular labeling, total H -irregularity strength, copies of graphs, union of graphs.

2010 Mathematics Subject Classification: 05C78, 05C70.

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Received 12 April 2017
Revised 5 February 2018
Accepted 22 February 2018