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ON THE MINIMUM NUMBER OF SPANNING TREES IN CUBIC MULTIGRAPHS

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Abstract

Let G_{2n}, H_{2n} be two non-isomorphic connected cubic multigraphs of order $2n$ with parallel edges permitted but without loops. Let $t(G_{2n}), t(H_{2n})$ denote the number of spanning trees in G_{2n}, H_{2n}, respectively. We prove that for $n \geq 3$ there is the unique G_{2n} such that $t(G_{2n}) < t(H_{2n})$ for any H_{2n}. Furthermore, we prove that such a graph has $t(G_{2n}) = 5^2 2^{n-3}$ spanning trees. Based on our results we give a conjecture for the unique r-regular connected graph H_{2n} of order $2n$ and odd degree r that minimizes the number of spanning trees.

Keywords: cubic multigraph, spanning tree, regular graph, enumeration.

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