

THE PROPER DIAMETER OF A GRAPH

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Abstract

A proper edge-coloring of a graph is a coloring in which adjacent edges receive distinct colors. A path is properly colored if consecutive edges have distinct colors, and an edge-colored graph is properly connected if there exists a properly colored path between every pair of vertices. In such a graph, we introduce the notion of the graph's proper diameter—which is a function of both the graph and the coloring—and define it to be the maximum length of a shortest properly colored path between any two vertices in the graph. We consider various families of graphs to find bounds on the gap between the diameter and possible proper diameters, paying singular attention to 2-colorings.

Keywords: diameter, properly connected, proper diameter.

2010 Mathematics Subject Classification: 05C12, 05C15.

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doi:10.20429/tag.2015.000102

Received 23 January 2017
Revised 15 February 2018
Accepted 15 February 2018