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ON THE METRIC DIMENSION OF DIRECTED AND UNDIRECTED CIRCULANT GRAPHS

Tomáš Vetrík

Department of Mathematics and Applied Mathematics
University of the Free State
Bloemfontein, South Africa

e-mail: vetrikt@ufs.ac.za

Abstract

The undirected circulant graph $C_n(\pm 1, \pm 2, \dots, \pm t)$ consists of vertices $v_0, v_1, \dots, v_{n-1}$ and undirected edges $v_i v_{i+j}$, where $0 \leq i \leq n-1$, $1 \leq j \leq t$ $(2 \leq t \leq \frac{n}{2})$, and the directed circulant graph $C_n(1, t)$ consists of vertices $v_0, v_1, \dots, v_{n-1}$ and directed edges $v_i v_{i+1}, v_i v_{i+t}$, where $0 \leq i \leq n-1$ $(2 \leq t \leq n-1)$, the indices are taken modulo n. Results on the metric dimension of undirected circulant graphs $C_n(\pm 1, \pm t)$ are available only for special values of t. We give a complete solution of this problem for directed graphs $C_n(1, t)$ for every $t \geq 2$ if $n \geq 2t^2$. Grigorious *et al.* [*On the metric dimension of circulant and Harary graphs*, Appl. Math. Comput. 248 (2014) 47–54] presented a conjecture saying that $dim\big(C_n(\pm 1, \pm 2, \dots, \pm t)\big) = t + p - 1$ for $n = 2tk + t + p$, where $3 \leq p \leq t + 1$. We disprove it by showing that $dim\big(C_n(\pm 1, \pm 2, \dots, \pm t)\big) \leq t + \frac{p+1}{2}$ for $n = 2tk + t + p$, where $t \geq 4$ is even, p is odd, $1 \leq p \leq t + 1$ and $k \geq 1$.

Keywords: metric dimension, resolving set, circulant graph, distance.

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