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BOUNDS ON THE LOCATING-TOTAL DOMINATION NUMBER IN TREES

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Abstract

Given a graph $G = (V, E)$ with no isolated vertex, a subset S of V is called a total dominating set of G if every vertex in V has a neighbor in S. A total dominating set S is called a locating-total dominating set if for each pair of distinct vertices u and v in $V \setminus S$, $N(u) \cap S \neq N(v) \cap S$. The minimum cardinality of a locating-total dominating set of G is the locating-total domination number, denoted by $\gamma_t^L(G)$. We show that, for a tree T of order $n \geq 3$ and diameter d, $\frac{d+1}{2} \leq \gamma_t^L(T) \leq n - \frac{d-1}{2}$, and if T has l leaves, s support vertices and s_1 strong support vertices, then $\gamma_t^L(T) \geq \max\left\{\frac{n+l-s+1}{2} - \frac{s+s_1}{4}, \frac{2(n+1)+3(l-s)-s_1}{5}\right\}$. We also characterize the extremal trees achieving these bounds.

Keywords: tree, total dominating set, locating-total dominating set, locating-total domination number.

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References

[1] M. Blidia and W. Dali, *A characterization of locating-total domination edge critical graphs*, Discuss. Math. Graph Theory **31** (2011) 197–202.
doi:10.7151/dmgt.1538

[2] M. Chellali, *On locating and differentiating-total domination in trees*, Discuss. Math. Graph Theory **28** (2008) 383–392.
doi:10.7151/dmgt.1414

[3] M. Chellali and N. Jafari Rad, *Locating-total domination critical graphs*, Australas. J. Combin. **45** (2009) 227–234.

[4] X. Chen and M.Y. Sohn, *Bounds on the locating-total domination number of a tree*, Discrete Appl. Math. **159** (2011) 769–773.
doi:10.1016/j.dam.2010.12.025

[5] C.J. Colbourn, P.J. Slater and L.K. Stewart, *Locating-dominating sets in series-parallel networks*, Congr. Numer. **56** (1987) 135–162.

[6] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, 1998).

[7] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, 1998).

[8] T.W. Haynes, M.A. Henning and J. Howard, *Locating and total dominating sets in trees*, Discrete Appl. Math. **154** (2006) 1293–1300.
doi:10.1016/j.dam.2006.01.002

[9] M.A. Henning and N. Jafari Rad, *Locating-total domination in graphs*, Discrete Appl. Math. **160** (2012) 1986–1993.
doi:10.1016/j.dam.2012.04.004

[10] N. Jafari Rad and H. Rahbani, *A note on the locating-total domination number in trees*, Australas. J. Combin. **66** (2016) 420–424.

[11] W.J. Ning, M. Lu and J. Guo, *Bounds on the differentiating-total domination number of a tree*, Discrete Appl. Math. **200** (2016) 153–160.
doi:10.1016/j.dam.2015.06.029

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