

DECOMPOSITIONS OF CUBIC TRACEABLE GRAPHS

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Abstract

A *traceable graph* is a graph with a Hamilton path. The 3-Decomposition Conjecture states that every connected cubic graph can be decomposed into a spanning tree, a 2-regular graph and a matching. We prove the conjecture for cubic traceable graphs.

Keywords: decomposition, cubic traceable graph, spanning tree, matching, 2-regular graph.

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REFERENCES

- [1] S. Akbari, T.R. Jensen and M. Siggers, *Decompositions of graphs into trees, forests, and regular subgraphs*, *Discrete Math.* **338** (2015) 1322–1327.
doi:10.1016/j.disc.2015.02.021
- [2] M.O. Albertson, D.M. Berman, J.P. Hutchinson and C. Thomassen, *Graphs with homeomorphically irreducible spanning trees*, *J. Graph Theory* **14** (1990) 247–258.
doi:10.1002/jgt.3190140212
- [3] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Springer, New York, 2008).
- [4] P.J. Cameron, *Research problems from the BCC22*, *Discrete Math.* **311** (2011) 1074–1083.
doi:10.1016/j.disc.2011.02.024

- [5] G.T. Chen, H. Ren and S.L. Shan, *Homeomorphically irreducible spanning trees in locally connected graphs*, *Combin. Probab. Comput.* **21** (2012) 107–111.
doi:10.1017/S0963548311000526
- [6] G.T. Chen and S.L. Shan, *Homeomorphically irreducible spanning trees*, *J. Combin. Theory Ser. B* **103** (2013) 409–414.
doi:10.1016/j.jctb.2013.04.001
- [7] J. Diemunsch, M. Furuya, M. Sharifzadeh, S. Tsuchiya, D. Wang, J. Wise and E. Yeager, *A characterization of P_5 -free graphs with a homeomorphically irreducible spanning tree*, *Discrete Appl. Math.* **185** (2015) 71–78.
doi:10.1016/j.dam.2014.12.023
- [8] R.J. Douglas, *NP-completeness and degree restricted spanning trees*, *Discrete Math.* **105** (1992) 41–47.
doi:10.1016/0012-365X(92)90130-8
- [9] G.H. Fan and A. Raspaud, *Fulkerson's conjecture and circuit covers*, *J. Combin. Theory Ser. B* **61** (1994) 133–138.
doi:10.1006/jctb.1994.1039
- [10] A. Hoffmann-Ostenhof, *Nowhere-Zero Flows and Structures in Cubic Graphs*, Ph.D. Dissertation (University of Vienna, 2011).
- [11] A. Hoffmann-Ostenhof, *A survey on the 3-decomposition conjecture* (2016), manuscript.
- [12] A. Hoffmann-Ostenhof, T. Kaiser and K. Ozeki, *Decomposing planar cubic graphs*, *J. Graph Theory* **88** (2018) 631–640.
doi:10.1002/jgt.22234
- [13] A. Kostochka, *Spanning trees in 3-regular graphs*, in: *REGS in Combinatorics* (University of Illinois at Urbana-Champaign, 2009).
- [14] J. Malkevitch, *Spanning trees in polytopal graphs*, *Ann. New York Acad. Sci.* **319** (1979) 362–367.
doi:10.1111/j.1749-6632.1979.tb32810.x
- [15] K. Ozeki and D. Ye, *Decomposing plane cubic graphs*, *European J. Combin.* **52** (2016) 40–46.
doi:10.1016/j.ejc.2015.08.005
- [16] J. Petersen, *Die Theorie der regulären graphen*, *Acta Math.* **15** (1891) 193–220.
doi:10.1007/BF02392606
- [17] V.G. Vizing, *On an estimate of the chromatic class of a p -graph*, *Metody Diskret. Anal.* **3** (1964) 25–30, in Russian.
- [18] D.B. West, *Introduction to Graph Theory* (Prentice-Hall, 2001).
- [19] D.Ye, (2016), personal communication.

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