CORRIGENDUM TO: BOUNDS ON THE NUMBER OF EDGES OF EDGE-MINIMAL, EDGE-MAXIMAL AND \(l\)-HYPERTREES [DISCUSSIONES MATHEMATICAE GRAPH THEORY 36 (2016) 259–278]

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Abstract

In this corrigendum, we correct the proof of Theorem 10 from our paper titled „Bounds on the number of edges of edge-minimal, edge-maximal and \(l\)-hypertrees”.

Keywords: hypertree, chain in hypergraph, edge-minimal hypertree, edge-maximal hypertree, 2-hypertree, Steiner system.

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1. The Corrected Proof of Theorem 10

In the original proof of Theorem 10, we stated that

\[
\frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)} |\mathcal{E}| \\
\leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)^2(n-k+1)} \binom{n}{k-1}.
\]

This can only be true if \(|\mathcal{E}| \geq \frac{1}{k-1} \binom{n}{k-1}\), but in reality, the exact opposite is true, i.e., \(|\mathcal{E}| \leq \frac{1}{k-1} \binom{n}{k-1}\) (see Theorem 9).

Below, we present the corrected proof of Theorem 10.

**Theorem 10.** If \(\mathcal{H} = (V, \mathcal{E})\) is a \(k\)-uniform 2-hypertree, then \(|\mathcal{E}| \leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)^2} \binom{n}{k-2}\).
**Proof.** We use the simple fact that $\sum_{i=1}^{n-k+1} C_i \geq \frac{1}{n-k+1} |\mathcal{E}|$, which follows from $|\mathcal{E}| = \sum_{i=1}^{n-k+1} i C_i \leq (n - k + 1) \sum_{i=1}^{n-k+1} C_i$.

Comparing it to the Star-equation (Theorem 9), we get

$$|\mathcal{E}| \leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)} |\mathcal{E}| - \frac{1}{k-1}!$$

$$\leq \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)(n-k+1)} |\mathcal{E}|,$$

which implies, that

$$|\mathcal{E}| \leq \left( (k-1) + \frac{1}{(n-k+1)} \right)^{-1} \binom{n}{k-1}$$

$$= \left( \frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+1) + (k-1)} \right) \binom{n}{k-1}$$

$$\leq \left( \frac{1}{k-1} - \frac{1}{(k-1)^2(n-k+2)} \right) \binom{n}{k-1}$$

$$= \frac{1}{k-1} \binom{n}{k-1} - \frac{1}{(k-1)^3} \binom{n}{k-2}.$$

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