Discussiones Mathematicae
Graph Theory 35 (2015) 271–281
doi:10.7151/dmgt.1795

ON *k*-PATH PANCYCLIC GRAPHS

Zhenming Bi and Ping Zhang

Department of Mathematics
Western Michigan University
Kalamazoo, MI 49008, USA

e-mail: ping.zhang@wmich.edu

Abstract

For integers k and n with $2 \leq k \leq n-1$, a graph G of order n is k-path pancyclic if every path P of order k in G lies on a cycle of every length from $k+1$ to n. Thus a 2-path pancyclic graph is edge-pancyclic. In this paper, we present sufficient conditions for graphs to be k-path pancyclic. For a graph G of order $n \geq 3$, we establish sharp lower bounds in terms of n and k for (a) the minimum degree of G, (b) the minimum degree-sum of nonadjacent vertices of G and (c) the size of G such that G is k-path pancyclic

Keywords: Hamiltonian, panconnected, pancyclic, path Hamiltonian, path pancyclic.

2010 Mathematics Subject Classification: 05C45, 05C38.

References

[1] Y. Alavi and J.E. Williamson, *Panconnected graphs*, Studia Sci. Math. Hungar. **10** (1975) 19–22.

[2] H.C. Chan, J.M. Chang, Y.L. Wang and S.J. Horng, *Geodesic-pancyclic graphs*, Discrete Appl. Math. **155** (2007) 1971–1978.

[3] G. Chartrand, F. Fujie and P. Zhang, *On an extension of an observation of Hamilton*, J. Combin. Math. Combin. Comput., to appear.

[4] G. Chartrand, A.M. Hobbs, H.A. Jung, S.F. Kapoor and C.St.J.A. Nash-Williams, *The square of a block is Hamiltonian connected*, J. Combin. Theory (B) **16** (1974) 290–292.
doi:10.1016/0095-8956(74)90075-6

[5] G. Chartrand and S.F. Kapoor, *The cube of every connected graph is* 1*-hamiltonian*, J. Res. Nat. Bur. Standards B **73** (1969) 47–48.
doi:10.6028/jres.073B.007

[6] G. Chartrand, L. Lesniak and P. Zhang, Graphs & Digraphs: 5th Edition (Chapman & Hall/CRC, Boca Raton, FL, 2010).

[7] G.A. Dirac, *Some theorems on abstract graphs*, Proc. London Math. Soc. **2** (1952) 69–81.
doi:10.1112/plms/s3-2.1.69

[8] R.J. Faudree and R.H. Schelp, *Path connected graphs*, Acta Math. Acad. Sci. Hungar. **25** (1974) 313–319.
doi:10.1007/BF01886090

[9] H. Fleischner, *The square of every two-connected graph is Hamiltonian*, J. Combin. Theory (B) **16** (1974) 29–34.
doi:10.1016/0095-8956(74)90091-4

[10] C.St.J.A. Nash-Williams, *Problem No.* 48, in: Theory of Graphs and its Applications, (Academic Press, New York 1968), 367.

[11] O. Ore, *Note on Hamilton circuits*, Amer. Math. Monthly **67** (1960) 55.
doi:10.2307/2308928

[12] O. Ore, *Hamilton connected graphs*, J. Math. Pures Appl. **42** (1963) 21–27.

[13] B. Randerath, I. Schiermeyer, M. Tewes and L. Volkmann, *Vertex pancyclic graphs*, Discrete Appl. Math. **120** (2002) 219–237.
doi:10.1016/S0166-218X(01)00292-X

[14] M. Sekanina, *On an ordering of the set of vertices of a connected graph*, Publ. Fac. Sci. Univ. Brno **412** (1960) 137–142.

[15] J.E. Williamson, *Panconnected graphs II*, Period. Math. Hungar. **8** (1977) 105–116.
doi:10.1007/BF02018497

Received 27 January 2014
Revised 25 June 2014
Accepted 25 June 2014