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GRAPHS WITH 4-RAINBOW INDEX 3 AND $n - 1$

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Abstract

Let G be a nontrivial connected graph with an edge-coloring $c : E(G) \to \{1, 2, \ldots, q\}$, $q \in \mathbb{N}$, where adjacent edges may be colored the same. A tree T in G is called a *rainbow tree* if no two edges of T receive the same color. For a vertex set $S \subseteq V(G)$, a tree that connects S in G is called an *S-tree*. The minimum number of colors that are needed in an edge-coloring of G such that there is a rainbow S-tree for every set S of k vertices of $V(G)$ is called the *k-rainbow index* of G, denoted by $rx_k(G)$. Notice that a lower bound and an upper bound of the k-rainbow index of a graph with order n is $k - 1$ and $n - 1$, respectively. Chartrand *et al.* got that the k-rainbow index of a tree with order n is $n - 1$ and the k-rainbow index of a unicyclic graph with order n is $n - 1$ or $n - 2$. Li and Sun raised the open problem of characterizing the graphs of order n with $rx_k(G) = n - 1$ for $k \geq 3$. In early papers we characterized the graphs of order n with 3-rainbow index 2 and $n - 1$. In this paper, we focus on $k = 4$, and characterize the graphs of order n with 4-rainbow index 3 and $n - 1$, respectively.

Keywords: rainbow S-tree, k-rainbow index.

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