G R A P H S   W I T H   4 - R A I N B O W   I N D E X   3   A N D   n − 1

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Abstract
Let G be a nontrivial connected graph with an edge-coloring c : E(G) → {1, 2, . . . , q}, q ∈ N, where adjacent edges may be colored the same. A tree T in G is called a rainbow tree if no two edges of T receive the same color. For a vertex set S ⊆ V(G), a tree that connects S in G is called an S-tree. The minimum number of colors that are needed in an edge-coloring of G such that there is a rainbow S-tree for every set S of k vertices of V(G) is called the k-rainbow index of G, denoted by r_x_k(G). Notice that a lower bound and an upper bound of the k-rainbow index of a graph with order n is k − 1 and n − 1, respectively. Chartrand et al. got that the k-rainbow index of a tree with order n is n − 1 and the k-rainbow index of a unicyclic graph with order n is n − 1 or n − 2. Li and Sun raised the open problem of characterizing the graphs of order n with r_x_k(G) = n − 1 for k ≥ 3. In early papers we characterized the graphs of order n with 3-rainbow index 2 and n − 1. In this paper, we focus on k = 4, and characterize the graphs of order n with 4-rainbow index 3 and n − 1, respectively.

Keywords: rainbow S-tree, k-rainbow index.

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