

## ETERNAL DOMINATION: CRITICALITY AND REACHABILITY

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### Abstract

We show that for every minimum eternal dominating set,  $D$ , of a graph  $G$  and every vertex  $v \in D$ , there is a sequence of attacks at the vertices of  $G$  which can be defended in such a way that an eternal dominating set not containing  $v$  is reached. The study of the stronger assertion that such a set can be reached after a single attack is defended leads to the study of graphs which are critical in the sense that deleting any vertex reduces the eternal domination number. Examples of these graphs and tight bounds on connectivity, edge-connectivity and diameter are given. It is also shown that there exist graphs in which deletion of any edge increases the eternal domination number, and graphs in which addition of any edge decreases the eternal domination number.

**Keywords:** dominating set, eternal dominating set, critical graphs.

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