

EXTENDING THE MAX ALGORITHM FOR MAXIMUM INDEPENDENT SET

NGOC C. LÊ^{1,2}, CHRISTOPH BRAUSE²

AND

INGO SCHIERMEYER²

¹ *School of Applied Mathematics and Informatics
Hanoi University of Science and Technology*

² *Faculty of Mathematics and Computer Science
Technische Universität Bergakademie Freiberg*

e-mail: lechingoc@yahoo.com
brause@math.tu-freiberg.de
ingo.schiermeyer@tu-freiberg.de

Abstract

The maximum independent set problem is an NP-hard problem. In this paper, we consider Algorithm MAX, which is a polynomial time algorithm for finding a maximal independent set in a graph G . We present a set of forbidden induced subgraphs such that Algorithm MAX always results in finding a maximum independent set of G . We also describe two modifications of Algorithm MAX and sets of forbidden induced subgraphs for the new algorithms.

Keywords: maximum independent set, stable set, stability number, independence number, reduction, graph transformation, MAX Algorithm, MIN Algorithm, Vertex Order Algorithm.

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