

A FINITE CHARACTERIZATION AND RECOGNITION  
OF INTERSECTION GRAPHS OF HYPERGRAPHS WITH  
RANK AT MOST 3 AND MULTIPLICITY AT MOST 2  
IN THE CLASS OF THRESHOLD GRAPHS

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**Abstract**

We characterize the class  $L_3^2$  of intersection graphs of hypergraphs with rank at most 3 and multiplicity at most 2 by means of a finite list of forbidden induced subgraphs in the class of threshold graphs. We also give an  $O(n)$ -time algorithm for the recognition of graphs from  $L_3^2$  in the class of threshold graphs, where  $n$  is the number of vertices of a tested graph.

**Keywords:** intersection graph, hypergraph rank, hypergraph multiplicity, forbidden induced subgraph, threshold graph.

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