

PRODUCTS OF DIGRAPHS AND THEIR COMPETITION GRAPHS

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Abstract

If $D = (V, A)$ is a digraph, its *competition graph (with loops)* $CG^l(D)$ has the vertex set V and $\{u, v\} \subseteq V$ is an edge of $CG^l(D)$ if and only if there is a vertex $w \in V$ such that $(u, w), (v, w) \in A$. In $CG^l(D)$, loops $\{v\}$ are allowed only if v is the only predecessor of a certain vertex $w \in V$. For several products $D_1 \circ D_2$ of digraphs D_1 and D_2 , we investigate the relations between the competition graphs of the factors D_1, D_2 and the competition graph of their product $D_1 \circ D_2$.

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REFERENCES

- [1] J. Bang-Jensen and G. Gutin, *Digraphs: Theory, Algorithms and Applications* (Springer, London, 2001).
- [2] J.E. Cohen, *Interval graphs and food webs: a finding and a problem* (Rand Corporation Document 17696-PR, Santa Monica, CA, 1968).
- [3] W. Imrich and S. Klavžar, *Product Graphs* (John Wiley & Sons, Inc., New York, 2000).

- [4] S.R. Kim, *The competition number and its variants*, in: Quo vadis, graph theory?, J. Gimbel, J.W. Kennedy and L.V. Quintas (Eds.), Ann. Discrete Math. **55** (1993) 313–326.
doi:10.1016/s0167-5060(08)70396-0
- [5] J.R. Lundgren, *Food webs, competition graphs, competition-common enemy graphs and niche graphs*, in: Applications of combinatorics and graph theory to the biological and social sciences, F. Roberts (Ed.), (IMA **17**, Springer, New York, 1989) 221–243.
doi:10.1007/978-1-4684-6381-1_9
- [6] F.S. Roberts, *Competition graphs and phylogeny graphs*, in: Graph theory and combinatorial biology, L. Lovász (Ed.), (Proc. Int. Colloqu. Balatonlelle (Hungary) 1996, Bolyai Soc. Math. Studies **7**, Budapest, 1999) 333–362.
- [7] M. Sonntag and H.-M. Teichert, *Competition hypergraphs*, Discrete Appl. Math. **143** (2004) 324–329.
doi:10.1016/j.dam.2004.02.010
- [8] M. Sonntag and H.-M. Teichert, *Competition hypergraphs of products of digraphs*, Graphs Combin. **25** (2009) 611–624.
doi:10.1007/s00373-005-0868-9

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