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SPLIT EULER TOURS IN 4-REGULAR PLANAR GRAPHS

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Abstract

The construction of a homing tour is known to be NP-complete. On the other hand, the Euler formula puts sufficient restrictions on plane graphs that one should be able to assert the existence of such tours in some cases; in particular we focus on split Euler tours (SETs) in 3-connected, 4-regular, planar graphs (tfps). An Euler tour S in a graph G is a SET if there is a vertex v (called a *half vertex* of S) such that the longest portion of the tour between successive visits to v is exactly half the number of edges of G. Among other results, we establish that every tfp G having a SET S in which every vertex of G is a half vertex of S can be transformed to another tfp G' having a SET S' in which every vertex of G' is a half vertex of S' and G' has at most one point having a face configuration of a particular class. The various results rely heavily on the structure of such graphs as determined by the Euler formula and on the construction of tfps from the octahedron. We also construct a 2-connected 4-regular planar graph that does not have a SET.

Keywords: 4-regular, 3-connected, planar, split Euler tour, NP-complete.

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