

## DENSE ARBITRARILY PARTITIONABLE GRAPHS

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### Abstract

A graph  $G$  of order  $n$  is called *arbitrarily partitionable* (AP for short) if, for every sequence  $(n_1, \dots, n_k)$  of positive integers with  $n_1 + \dots + n_k = n$ , there exists a partition  $(V_1, \dots, V_k)$  of the vertex set  $V(G)$  such that  $V_i$  induces a connected subgraph of order  $n_i$  for  $i = 1, \dots, k$ . In this paper we show that every connected graph  $G$  of order  $n \geq 22$  and with  $\|G\| > \binom{n-4}{2} + 12$  edges is AP or belongs to few classes of exceptional graphs.

**Keywords:** arbitrarily partitionable graph, Erdős-Gallai condition, traceable graph, perfect matching.

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