KERNELS BY MONOCHROMATIC PATHS AND COLOR-PERFECT DIGRAPHS

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Abstract

For a digraph \( D \), \( V(D) \) and \( A(D) \) will denote the sets of vertices and arcs of \( D \) respectively. In an arc-colored digraph, a subset \( K \) of \( V(D) \) is said to be kernel by monochromatic paths (mp-kernel) if (1) for any two different vertices \( x, y \) in \( N \) there is no monochromatic directed path between them (\( N \) is mp-independent) and (2) for each vertex \( u \) in \( V(D) \) \( \setminus N \) there exists \( v \in N \) such that there is a monochromatic directed path from \( u \) to \( v \) in \( D \) (\( N \) is mp-absorbent). If every arc in \( D \) has a different color, then a kernel by monochromatic paths is said to be a kernel. Two associated digraphs to an arc-colored digraph are the closure and the color-class digraph \( C_C(D) \).

In this paper we will approach an mp-kernel via the closure of induced subdigraphs of \( D \) which have the property of having few colors in their arcs with respect to \( D \). We will introduce the concept of color-perfect digraph and we are going to prove that if \( D \) is an arc-colored digraph such that \( D \) is a quasi color-perfect digraph and \( C_C(D) \) is not strong, then \( D \) has an mp-kernel. Previous interesting results are generalized, as for example Richardson’s Theorem.

**Keywords:** kernel, kernel perfect digraph, kernel by monochromatic paths, color-class digraph, quasi color-perfect digraph, color-perfect digraph.

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