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# SOLUTIONS OF SOME $L(2,1)$-COLORING RELATED OPEN PROBLEMS

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**Abstract**

An $L(2,1)$-*coloring* (or *labeling*) of a graph $G$ is a vertex coloring $f : V(G) \to Z^+ \cup \{0\}$ such that $|f(u) - f(v)| \geq 2$ for all edges $uv$ of $G$, and $|f(u)-f(v)| \geq 1$ if $d(u,v) = 2$, where $d(u,v)$ is the distance between vertices $u$ and $v$ in $G$. The *span of an* $L(2,1)$-*coloring* is the maximum color (or label) assigned by it. The *span of a graph* $G$ is the smallest integer $\lambda$ such that there exists an $L(2,1)$-coloring of $G$ with span $\lambda$. An $L(2,1)$-coloring of a graph with span equal to the span of the graph is called a *span coloring*. For an $L(2,1)$-coloring $f$ of a graph $G$ with span $k$, an integer $h$ is a *hole* in $f$ if $h \in (0,k)$ and there is no vertex $v$ in $G$ such that $f(v) = h$. A *no-hole coloring* is an $L(2,1)$-coloring with no hole in it. An $L(2,1)$-coloring is *irreducible* if color of none of the vertices in the graph can be decreased to yield another $L(2,1)$-coloring of the same graph. A graph $G$ is *inh-colorable* if there exists an irreducible no-hole coloring of $G$. Most of the results obtained in this paper are answers to some problems asked by Laskar *et al.* [5]. These problems are mainly about relationship between the span and maximum no-hole span of a graph, lower inh-span and upper inh-span of a graph, and the maximum number of holes and minimum number of holes in a span coloring of a graph. We also give some sufficient conditions for a tree and an unicyclic graph to have inh-span $\Delta + 1$.

**Keywords:** $L(2,1)$-coloring, span of a graph, no-hole coloring, irreducible coloring, unicyclic graph.

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