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VERTICES CONTAINED IN ALL OR IN NO MINIMUM SEMITOTAL DOMINATING SET OF A TREE

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Abstract

Let G be a graph with no isolated vertex. In this paper, we study a parameter that is squeezed between arguably the two most important domination parameters; namely, the domination number, $\gamma(G)$, and the total domination number, $\gamma_t(G)$. A set S of vertices in a graph G is a semitotal dominating set of G if it is a dominating set of G and every vertex in S is within distance 2 of another vertex of S. The semitotal domination number, $\gamma_{t2}(G)$, is the minimum cardinality of a semitotal dominating set of G. We observe that $\gamma(G) \leq \gamma_{t2}(G) \leq \gamma_t(G)$. We characterize the set of vertices that are contained in all, or in no minimum semitotal dominating set of a tree.

Keywords: domination, semitotal domination, trees.

2010 Mathematics Subject Classification: 05C69.

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Appendix

We now present an example to illustrate Theorem **??**. Applying our pruning process discussed in Section **??** to the rooted tree T with root v illustrated in Figure 1(a), we proceed as follows.

• The branch vertices b_3 and b_4 are both at maximum distance 3 from v in T. We select b_3, where $|L^3(b_3)| = 1$. Thus, b_3 is a type-(T.1) branch vertex and we delete $D(b_3)$ and attach a path of length 3 to b_3.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(b)) is the vertex b_4. Since $|L^1(b_4)| > 2$ and every leaf-descendant of b_4 belongs to $L^1(b_4)$, the vertex b_4 is therefore a type-(T.3) branch vertex and we delete $D(b_4)$ and attach a path of length 1 to b_4.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(c)) is the vertex b_2. Since $|L^4(b_2)| = 1$ and $L^1(b_2) = L^3(b_2) = \emptyset$, the vertex b_2 is a type-(T.4) branch vertex and we delete $D(b_2)$ and attach a path of length 4 to b_2.

• The branch vertex at maximum distance from v in the resulting tree (illustrated in Figure 1(d)) is the vertex b_1. Since $|L^3(b_1)| = 1$, the vertex b_1 is a type-(T.1) branch

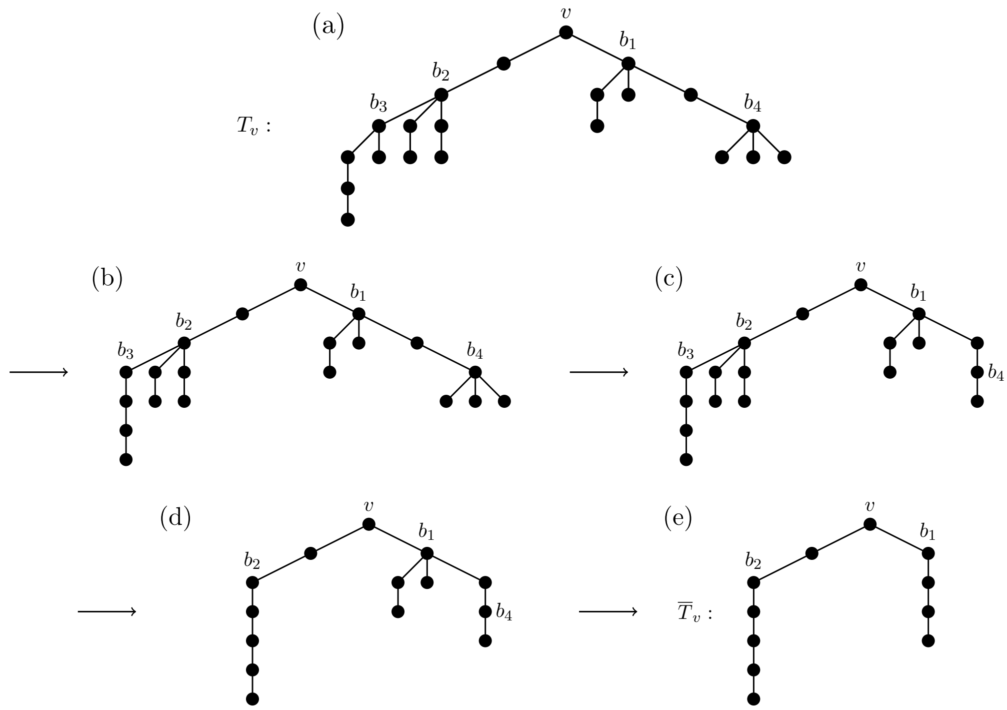

Figure 1.The pruning of a tree rooted at v.

vertex and we delete $D(b_1)$ and attach a path of length 3 to b_1. The resulting pruned tree $\overline{T}_v$ is illustrated in Figure 1(e).

• Since $|\overline{L}^1(v)| = 1$ and $|\overline{L}^4(v)| = 1$, by Theorem **??**, we deduce that $v \notin \mathcal{A}_{t2}(T) \cup \mathcal{N}_{t2}(T)$.

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