

## A NOTE ON THE TOTAL DETECTION NUMBERS OF CYCLES

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### Abstract

Let  $G$  be a connected graph of size at least 2 and  $c : E(G) \rightarrow \{0, 1, \dots, k-1\}$  an edge coloring (or labeling) of  $G$  using  $k$  labels, where adjacent edges may be assigned the same label. For each vertex  $v$  of  $G$ , the color code of  $v$  with respect to  $c$  is the  $k$ -vector  $\text{code}(v) = (a_0, a_1, \dots, a_{k-1})$ , where  $a_i$  is the number of edges incident with  $v$  that are labeled  $i$  for  $0 \leq i \leq k-1$ . The labeling  $c$  is called a detectable labeling if distinct vertices in  $G$  have distinct color codes. The value  $\text{val}(c)$  of a detectable labeling  $c$  of a graph  $G$  is the sum of the labels assigned to the edges in  $G$ . The total detection number  $\text{td}(G)$  of  $G$  is defined by  $\text{td}(G) = \min\{\text{val}(c)\}$ , where the minimum is taken over all detectable labelings  $c$  of  $G$ . We investigate the problem of determining the total detection numbers of cycles.

**Keywords:** vertex-distinguishing coloring, detectable labeling, detection number, total detection number, Hamiltonian graph.

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