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$(k-1)$-KERNELS IN STRONG k-TRANSITIVE DIGRAPHS

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Abstract

Let $D = (V(D), A(D))$ be a digraph and $k \geq 2$ be an integer. A subset N of $V(D)$ is k-independent if for every pair of vertices $u, v \in N$, we have $d(u, v) \geq k$; it is l-absorbent if for every $u \in V(D) - N$, there exists $v \in N$ such that $d(u, v) \leq l$. A (k, l)-kernel of D is a k-independent and l-absorbent subset of $V(D)$. A k-kernel is a $(k, k-1)$-kernel.

A digraph D is k-transitive if for any path $x_0x_1 \cdots x_k$ of length k, x_0 dominates x_k. Hernández-Cruz [3-*transitive digraphs*, Discuss. Math. Graph Theory **32** (2012) 205–219] proved that a 3-transitive digraph has a 2-kernel if and only if it has no terminal strong component isomorphic to a 3-cycle. In this paper, we generalize the result to strong k-transitive digraphs and prove that a strong k-transitive digraph with $k \geq 4$ has a $(k-1)$-kernel if and only if it is not isomorphic to a k-cycle.

Keywords: digraph, transitive digraph, k-transitive digraph, k-kernel.

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