

## SOME PROGRESS ON THE DOUBLE ROMAN DOMINATION IN GRAPHS

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### Abstract

For a graph  $G = (V, E)$ , a double Roman dominating function (or just DRDF) is a function  $f : V \rightarrow \{0, 1, 2, 3\}$  having the property that if  $f(v) = 0$  for a vertex  $v$ , then  $v$  has at least two neighbors assigned 2 under  $f$  or one neighbor assigned 3 under  $f$ , and if  $f(v) = 1$ , then vertex  $v$  must have at least one neighbor  $w$  with  $f(w) \geq 2$ . The weight of a DRDF  $f$  is the sum  $f(V) = \sum_{v \in V} f(v)$ , and the minimum weight of a DRDF on  $G$  is the double Roman domination number of  $G$ , denoted by  $\gamma_{dR}(G)$ . In this paper, we derive sharp upper and lower bounds on  $\gamma_{dR}(G) + \gamma_{dR}(\overline{G})$  and also  $\gamma_{dR}(G)\gamma_{dR}(\overline{G})$ , where  $\overline{G}$  is the complement of graph  $G$ . We also show that the decision problem for the double Roman domination number is NP-complete even when restricted to bipartite graphs and chordal graphs.

**Keywords:** Roman domination, double Roman domination.

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